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BOSTON UNIVERSITY
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Thesis

METHODS OF TEACHING ARITHMETIC
A Partial Summary of Available Studies

by
Mildred Browning Stone
(B.S. in Ed., Boston University, 1927)

submitted in partial fulfilment of the
requirements for the degree of
MASTER OF ARTS
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INTRODUCTION

Problem and Limits.

The purpose of this thesis is to summarize available studies on Methods of teaching arithmetic, except that studies on problem solving and remedial measures are excluded. Some summaries are accompanied by critical estimates.

Procedure.

The procedure used in this study includes (1) the selection of those studies whose titles in any way suggest that their content deals with Methods of teaching the subject-matter of arithmetic, (2) the preparation of summaries of those studies whose content relates directly to Methods of teaching arithmetic, and (3) the inclusion of brief statements of the nature of those studies whose content, contrary to the suggestion of their titles, does not contribute directly to teaching Methods.

Scope.

The difficulty of strictly limiting the scope of this summary to the field of Methods has led to a more or less arbitrary selection of studies to be reported. The materials which are to be treated in this summary include special studies of the teaching of subject-matter topics, followed by studies pertaining to motivation, drill, and transfer of training.

As discussion of arithmetic problems in general were encountered note was made of the topics considered by

others as phases of Methods. Stone[1] in 1913 mentions addition, subtraction, multiplication, division, fractions, denominate numbers, decimals, percentage, interest computation, reasoning, vacation losses, utilizing play tendencies, and distribution of arithmetic among grades. Buswell and Judd[2] in 1925 in a chapter on Methods of teaching include general books on methods, summaries of principles, methods of primary teaching, methods involving practical applications, methods relating to drill, incidental teaching, methodological suggestions based on theory, methodological suggestions based on experience, methodological principles based on experiments and tests. Other chapters are devoted to drill, individual differences and remedial methods, and transfer of training. West, Green and Brownell[3] in a chapter on "changing methods of instruction" list the place of objective teaching, larger use of pupil's environment, increased provision for self-activity, correlation, special methods of instruction (Grube, spiral, topical, problem), place of drill, games and devices, types of problems, use of textbook as a phase of methods, and providing for individual differences.

---

[1] Stone, C.W. Problems in the Scientific Study of the Teaching of Arithmetic. Journal of Educational Psychology 4:1-16. January 1913.

[2] Buswell, G.T. and Judd, C.H. Summary of Educational Investigations Relating to Arithmetic. Chapter 8. University of Chicago. 1925.

[3] West, R.L., Greene, Chas. E., and Brownell, W.A. The Arithmetic Curriculum. Twenty-ninth Yearbook of the National Society for the Study of Education 65-142. 1930.

The topics chosen for this study correspond reasonably well with the topics noted above.

The overlapping of problems of Curriculum and of Methods, due to the close relation between content to be taught and manner in which it is to be taught, is especially evident in the consideration of the place of informal and formal development of number in the primary grades. Not only does the amount of informal number to be included in the primary grades depend upon the knowledge possessed by teachers and administrators as to the results obtained through that method, but the execution of an informal program lies with the teacher. It has seemed wise therefore to consider this a question of Methods. With this exception curriculum studies have been avoided.

The teaching of problem solving is without any question a phase of Methods but the quantity of material in that field warrants its separate treatment. This is likewise true of studies pertaining to diagnostic and remedial teaching. An attempt has been made to exclude articles concerning the evaluation of text-books and drill devices and the results of standardized tests.

Unpublished theses for master's and doctor's degrees have been read whenever available.

No attempt has been made to limit this summary to those studies which have been made within a definite period of time. However the range of time represented by the materials included is from 1911 through the first six months of 1932. The latest studies reported here occurred in magazines

issued in June 1932.

Classification.

The materials in this summary classified as to content include (1) special studies of the teaching of subject-matter topics. viz. development of number concept, formal or informal nature of primary number, reading and writing of numbers, the relative difficulty of the number facts, addition, subtraction, multiplication, division, common fractions, decimal fractions, denominate numbers, percentage, checking computation, and (2) general phases of Methods, viz. motivation, drill, and transfer of training.

Within each content unit a further classification is made as to the type of study being summarized. The following criteria as a basis for classification were set up in Dr. Wilson's seminar course in 1931-1932:

a. Major Research

A profitable technique in any case.

In methods not less than twenty rooms for experimental and twenty for control purposes, over an appropriate period of time. Experiment repeated. Case studies or intensive room studies, a smaller number acceptable.

In curriculum not less than 1000 cases.

b. Minor Research

A profitable technique.

In methods and curriculum less than the above but large enough to avoid mere chance or the influence of irrelevant factors.

A control group.

c. Library Summary

Summaries of library materials based chiefly on primary not secondary sources.

d. Discussions

Didactic, argumentative, and otherwise.

In actual classification, however, it has not been possible to follow this set up for these reasons:

1. Only 34 studies report the use of control groups and of these only 11 used as many as twenty control groups.

2. Since a minor research, differing from a major research primarily in the number of subjects involved, also requires controlled conditions there is no classification provided for the vast majority of studies forming the bulk of material in this thesis.

3. In the number of pupils or subjects involved in a study the range is from 1 to 11,368. The various studies involve the technique of counting of responses given by each child to each number fact taught, the pre-test and re-test plan, the oral question and answer method as well as other common forms of investigation. They vary in scope from a limited study of a few facts or a single process step to comprehensive investigations of number facts in all processes and to examination of several related phases of Methods. They vary in the time over

which the experiment continued from a few hours to more than a year.

Therefore it has been necessary to classify these studies as major or minor researches on the basis of the number of subjects, scope of investigation, and duration of time, on a less objective basis than that suggested by the foregoing criteria. However the writer feels that no great injustice has been done by the attempt to distinguish the studies of greater worth from those of lesser value.

Library summaries and discussions have been classified with little difficulty.

The order of the studies, under each classification as to type, is chronological making possible an acquaintance by the reader with recent materials by consulting the last studies listed.

Sources.

The most helpful references to sources have been:

1. United States Bureau of Education Bulletins - Bibliography of Research Studies in Education: 1928, #22; 1929,#36; 1930, #23; 1931, #13.

2. Buswell, G. T. and Judd, C. H. Summary of Investigations Relating to Arithmetic. Supplementary Educational Monograph #27, Chicago University 1925. Supplements 1, 2, 3, 4, and 5, continuations of Monograph #27 are reprints from the Elementary School Journal as follows: #1 from Vol. 26, May and June 1926; #2 from Vol. 27, May and June 1927; #3 from Vol. 28, May and June 1928; #4 from Vol. 29, May and June 1929; and #5 from Vol. 30, June 1930. Vol. 31, June 1931 and Vol. 32,

June 1932 contain further summaries.

3. Pyle, W. H. The Psychology of the Common Branches. Chapters 10 to 16. Warwick and York 1930.

4. Rubado, C. A. Problems of the City School Superintendent in the Field of Arithmetic. Teachers College Contribution, #406. 1930.

Chapter One

THE DEVELOPMENT OF THE NUMBER CONCEPT

MAJOR RESEARCH

Judd, Charles Hubbard
Psychological Analysis of the Fundamentals of Arithmetic. Supplementary Educational Monograph #32. Chicago. Department of Education Chicago University. February 1927.

The 15 pages of chapter 1 of this monograph are devoted to the discussion of (1) experiments for the reader in preparation for the research described and (2) the development of our number system. Chapter 2 describes the apparatus and procedure and amply illustrates with tables (10) the results from counting experiments in which adults counted sounds and flashes which were varied in rate and number. These conclusions follow: "There are great individual differences in the ability to count. A given individual exibits wide differences in ability in counting different types of experiences. The competency of any individual in counting series of experiences more than ten in number is much less than his competency in counting short series - due in very large measure to the complicated number names which are used in counting beyond ten. Counting is a process consisting in a succession of inner acts which are, in essence, inner articulation of number names."

Chapter 3 describes similar procedure with 20 children from each of the grades from one to six, one-half

of the children being chosen from those judged by their teachers to be the most competent, the other half from those judged by their teachers to be less competent. Marked individual differences were found as in adult counting. There was steady progress throughout the elementary grades in ability to count series of sounds. Counting flashes of light also showed steady progress which however was far behind that for counting sounds. Observations made by teachers while children were counting are fully reported in an accompanying table. From these observations it seems that counting is an active process with lip movements, movements of the head or other parts of the body becoming gradually reduced until these reactions become slight inner tendencies to articulate.

Chapter 4 contains analyses of individual cases of 20 graduate students in experiments similar to those previously used together with introspective records kept throughout the experiment. There were found variations in accuracy ranging from 7 - 41 errors in counting number of sounds and from 3 - 19 in counting flashes of light.

In Chapter 5 the author discusses the apprehension of large numbers indirectly through analysis into smaller groups; the older method of teaching number combinations as tables versus the present less systematic method; the theory of specific training versus the theory of generalized systematic training; types of problems found in four texts in which last discussion the author concludes that "pupils are given little help in discovering the character of the underlying logical principles on which the arithmetof the children being studied. Two thirds proved by their attitude to be the most consistent, the other third were shown by their attitude to be less consistent. [illegible] were found [illegible] ing. Those who [illegible] throughout the elementary grades in addition to [illegible] of schools. [illegible] absence of [illegible] showed steady progress [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]

Chapter 4 contains analysis of individual cases of [illegible] [illegible] [illegible] number of [illegible] [illegible] of truth.

In Chapter 5 the author discusses the experimental [illegible] ical operations depend."

Chapter 6 outlines the author's psychology of the fundamentals of arithmetic. This psychology emphasizes mind training rather than preparation for needs of practical living.

Comment: The methods of investigation of reading are applied to numbers with little other than negative results. The author's conclusions go beyond the data.

Brownell, William A.
The Development of Children's Number Ideas in the Primary Grades.
Supplementary Educational Monograph #35. Chicago. Department of Education, Chicago University. August 1928.

The author of this research describes tests given to 1850 children in the first seven grades in eight schools in three towns in Illinois. His first problem was to determine the relative difficulty of apprehension of the numbers from 3 - 12 when these numbers were presented in visual concrete form. The tests given comprised six sets of exposure cards with dots arranged in the following forms: quadratic, diamond, domino, triangular, odd, linear. The pupil's response was to estimate the digit except in the first grade where the children drew what they saw.

Eleven tables show the author's findings, also the work done by Howell with whose work the author compares his own.

The author's conclusion in regard to this question

is that with the grouping factor controlled the difficulty of apprehension is in proportion to the size of the number, no numbers whether even or odd being inherently easy or hard.

A second question concerned the changes in the methods of apprehension in the first five grades. Conclusions are based on inferences from results of tests used in the previous problem, results from questioning pupils, observation of methods during testing, and children's number pictures drawn from memory.

Conclusions are that (1) children in the first three grades do not readily apprehend visual concrete numbers exposed to them in the form of number pictures; (2) children below grades 3 and 4 do not employ counting by 2's, 3's and 4's although they have met these counting methods in grade two; (3) a few pupils in grades four and five continue counting four. On the basis of these conclusions the author suggests the use of number pictures for teaching addition combinations, i.e. the counting of dots, etc.

In a third problem, the determination of factors in the development of ability to comprehend concrete numbers, the author after computation of statistical correlations concludes that in addition to mental age and amount of instruction there are other factors not clear to him. From an analysis of individual records he finds that children employ methods of counting, partial counting, grouping, and multiplication and conversion, for apprehending concrete visual number. The typical pupil employs several steps. No signifieant correlation is found between accuracy of apprehension and chronological age and but slightly posiis great with the concrete factor constant, the difference of improvement is in proportion to the size of the groups, [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible].

A second [illegible] concerned the changes in the methods of [illegible] in the [illegible] [illegible] grades. [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] of [illegible] [illegible] in the [illegible] [illegible] [illegible] [illegible] [illegible], [illegible] of [illegible] [illegible] [illegible], and [illegible] [illegible] [illegible] [illegible] [illegible].

[illegible] [illegible] [illegible] (1) [illegible] in the first three grades, [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] in the [illegible] of [illegible] [illegible] (2) [illegible]. In the [illegible] of [illegible] [illegible] the [illegible] [illegible] the use of [illegible] [illegible] [illegible] [illegible] [illegible] [illegible], i.e. the completion of ideas, etc.

Dr. [illegible], in [illegible], the determination of factors in the development of ability to [illegible] [illegible] [illegible], [illegible] [illegible] [illegible] [illegible] of [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible] the analysis of [illegible], [illegible] [illegible] that [illegible] [illegible] [illegible] of [illegible], [illegible] [illegible], [illegible], and [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]. [illegible]

tive correlation with mental age. While mature methods do not imply accuracy in the use of the same it is found in general that mature methods were employed by children with high I.Q.s who also obtained accuracy in comprehension.

A further problem to determine the mental processes of children in grades two A, three A, and four A, in learning additions combinations up to 9 and 9, was based on individual analysis. Dangerous practices which were discovered include reversing digits, i.e. knowing 6 and 9 only by reversing 9 + 6; habituating a solution for a combination, e.g. 7 + 7 = 14 + 1 = 15 for 7 + 8; attaching special preferences to certain combinations, e.g. the use of 9 + 5 and 8 + 4 to solve 9 + 6 and 8 + 5. The conclusion is that the change from concrete to abstract number is the point where currect methods are inadequate.

A final problem, to determine how children in grades three A and four A add three digits, concludes that in making the first additions pupils have difficulty with addition combinations because they do not understand concrete numbers. In making the second addition lack of understanding of additive combinations handicaps column addition.

The author's final chapter is a discussion of the teaching of number in the primary grades. He thinks that arithmetic as now taught is too difficult for a great many children. He criticizes the practice of telling the children the answer instead of letting them count. He also finds a failure to provide adequately for transfer of training.
Comment:The research student may look in vain for adequate data to support the author's views in regard to the teaching

of number in the primary grades.

Maclatchy, Josephine
A Phase of First Grade Readiness.
Educational Research Bulletin (Ohio State University) 10: 337-380. October 14, 1931.

This research involved 1242 six-year old children entering grade one at Cincinnati. Results are given for the lowest quarter, middle 50%, and highest quarter as determined from the Pintner-Cunningham Primary Mental Test.

In rote counting only 2% could not count beyond 4; 33% of the lowest quarter, 8% of the middle half and 4% of the highest quarter could not count beyond 9.

The ability to arrange and name five objects in one out of three trials was possessed by 86% of the highest quarter, 80% of the middle half and 50% of the lowest quarter. The ability to name five objects was possessed by 80% of the highest quarter, 77% of the middle half and 45% of the lowest quarter. The ability to arrange ten objects was possessed by 77% of the highest quarter, 59% of the middle half and 22% of the lowest quarter. The ability to name ten objects was possessed by 64% of the highest quarter, 45% of the middle half and 16% of the lowest quarter.

Ten addition combinations were used in this test. 5 + 1 was known by 1/3 of the lowest group, 2/3 of the middle group and 5/6 of the highest group. 4 + 5 was the least familiar of the combinations tested. The medians for combinations correct were 1.5, 4.8, and 6.8 respectively for the

lowest, middle and highest groups.

Miss Maclatchy says of her findings that they are "no argument for school bred arithmetic in the first grade but for child instruction rather than group instruction and for the continued use of the method by which he acquired these facts, adapted to school situations of course."

Maclatchy, Josephine H.
Counting and Addition.
Educational Research Bulletin (Ohio State University) 11: 96-100. February 17, 1932.

This study involved 708 six-year old children just entering the first grade at Cincinnati from the kindergartens of that city. Ten addition combinations were presented in the form: "If you have --- and get --- more, how many will you have then?" These ten combinations which had sums ten or less were: 5 + 1, 7 + 1, 1 + 9, 4 + 4, 1 + 6, 5 + 2, 8 + 2, 4 + 5, 5 + 3, 3 + 5.

One table shows the rote-counting ability of the children who knew the sums of none, one, two, etc., addition combinations. Of the 51 children who did not get any sums right, one counted to 100, the others not counting beyond 39 with a median interval of counting 10 to 14. Of the 57 children who knew the sums of all ten combinations, 49% counted to 100 or beyond, the lowest 5% counting only between 10 and 14. From this table it can be seen that as the number of correct combinations increases the percentage counting increases. This evidence is strengthened in the

results of a second subtest, a test of counting objects. The twenty objects provided were counted correctly by 72% of the children. The ability to count objects possessed by the group which knew the sums of none of the combinations and by the group which knew the sums of ten of the combinations is different as is now shown:

	Combinations correct	
	none Percent	ten Percent
Could not count	2	. .
Could not count beyond 5	16	. .
Could not count beyond 10	26	. .
Could not count beyond 15	50	6
Could count to 20	34	94

From her data the author points out the indication of a concomitant relation between counting and early familiarity with addition. The author goes on to remind us that these children have besides their informal practice in kindergarten received their instruction from family and playmates. Their facility in number has resulted from replies to their own questions, trick knowledge which their parents wished to impart, experience gained from games, toys, construction, buying at the store, selling papers and the like.

Comment: The data in both of these studies by Miss Maclatchy point to the amount of knowledge resulting from informal procedures.

results of a general nature, a type of knowledge frequently [illegible] of the [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] by the group which knew the names of none of the constellations and by the group which knew the names of one of the constellations is different to little degree.

[illegible]	[illegible]	[illegible]
Could not name	8	. .
Could not name correctly	20	. .
Could name [illegible]	40	. .
Could name [illegible]	40	6
[illegible]	24	[illegible]

From the data the author points out the importance of [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] writing, music and the like.

Comment: The data in both of these studies by Miss Englehart point to the amount of knowledge resulting from informal procedures.

MINOR RESEARCH

Douglass, H. R.

Development of Number Concept in Children of Pre-school and Kindergarten Ages.

Journal of Experimental Psychology 8: 443-70. December 1925.

Two classes of kindergarten children numbering 26 and 40 and ranging in ages from 4 years 6 months to 6 years were tested for their concepts of the numbers from 1 to 10. A dot recognition test was given three times - twice at the same sitting and again a month later. Dot selection and marble tests were given once each. A plan for weighting estimates was devised to measure concepts of whatever accuracy beginning with the concept not sufficiently formed to enable the possessor to make a better estimate than by chance.

Data are presented in 15 tables.

The author finds that children have comparatively accurate concepts of 1 and 2, very serviceable and accurate concepts of 3 and a very serviceable concept of 4. Concepts of 5, 6, 7, 8, 9, and 10 are rather vague although serviceable to a slight degree.

LIBRARY SUMMARY

Freeman, F. N.

Grouped Objects as a Concrete Basis for the Number Idea.

Elementary School Teacher 12:306-14. March 1912.

This article reports results of an experiment the original account of which is in German. Grouped and ungrouped dots were exposed on a screen before the observers who were children from 8 to 14 and adults.

Less difference in scope of attention was found than had been expected. Up to and including the number 4 younger children could judge as well as adults. There was a marked difference in the ability to perceive and correctly judge grouped objects which extended considerably beyond the scope of attention. Adults and older children grouped objects and observed the grouping of objects already grouped. The author concludes that counting is not the complete number idea but that grouping must be added. He suggests the use of grouped objects as a concrete basis for the idea of fundamental number operations.

DISCUSSION

Court, Sophie Ravitch Altshiller

Numbers, Time and Space in the First Five Years of a Child's Life.

Pedagogical Seminary 27:71-89. March 1920.

This article relates the development of numbers, time and space from the age of twenty months to five years in the life of a boy with an I.Q. of 135. The recognition of 2 came before 1, with the ability at twenty months to point out objects, name them, and look for mates . During the third year this child developed the ability to count by rote to 10, but his ability to count objects was limited to one, two, many. During the fourth year both rote and rational counting were extended to 50, adding and subtracting within 5 were acquired, fairly well without counters and excellently with, halves were accorded much interest, and dominoes were matched with enjoyment. During the fifth year the symbols 1 through 5 were learned in summer school, counting was

extended to 1000, the calendar was studied, the fingers were used abstractly, and answers were written to typed examples such as 1 + 1, 1 + 2, 1 + 3.

Concepts of time began with today and yesterday, later extending to morning, noon, afternoon and evening. The passing of the 6.30 train, the use of an old alarm clock, the answers to questions as "How long is a minute?" added to these time concepts.

Concepts of space began with long and short at two years and ten months. From that age until the age of three years and six months the child played carpenter, measuring tables for oilcloth. Interest was shown in house plans, a reflection of the family interest in the building of a house.

Court, Sophie Ravitch Altshiller
Self Taught Arithmetic from the Age of Five to the Age of Eight.
Pedagogical Seminary 30: 51-68. March 1923.

In this article is continued the narration of the development of concepts of number in the life of the child whose early number history is related in the preceding report.

In Table 1 new concepts have been classified to include (1) information obtained by his own discoveries or through persistent inquiries, subdivided into spontaneous intense and lasting interests and spontaneous but passing interests and (2) information gained through outside influence subdivided into those calling forth intense and lasting interest and those in which the interest was but momentary.

Table 2 divides the number concepts gained into these

classes: (1) those which the child would have learned in the first four grades including counting by 1, 2, 3, 5, and 10, reading numbers of two or three digits, writing figures, computation, doubling small numbers, the significance of zero, odd and even numbers; (2) those which could not have been learned in the first four grades including telling time, length of a minute, geometric figures, negative quantities, ratio, the duo-decimal system, and a test of long division by addition and (3) those which the child would have learned less thoroughly without the observation and practice which he actually had including measuring, halves, quarters, thirds and musical measures.

From Table 3 which lists over 30 concepts are taken the following and the age at which each made its appearance:

5 yr. to 5 yr. 6 mo. adding columns on the typewriter
5 yr. 6 wk. counting by fives
5 yr. 2 mo. measuring depths of liquids
5 yr. 2½ mo. odd and even numbers
5 yr. 3 mo. telling time
5 yr. 3 mo. playing store
5 yr. 4 mo. reading three digit numbers
5 yr. 4 mo. columns in writing
5 yr. 5 mo. $0 + 0 = 0$
5 yr. 5 mo. counting by 2, 5, and 10
5 yr. 5 mo. to 5 yr. 9 mo. adding on number blocks
6 yr. reading the thermometer
6 yr. 1 mo. 3000 and 30000
6 yr. 4 mo. geometric figures
6 yr. 6 mo. musical measures
6 yr. 7 mo. forever and a day
6 yr. 8½ mo. 6 below zero
7 yr. 7 mo. negative quantities
7 yr. 7 mo. 12345679 x 9
7 yr. 9 mo. ratio (months to years)
7 yr. 9½ mo. newwlong division proof
8 yr. 5 mo. decimal and duo-decimal systems

It is impossible in any summary of material of this kind to indicate the natural and spontaneous manner in which these number concepts were acquired by the child being

studied. The author points out frequently that there was no coercion on the part of the parents.

SUMMARY OF CHAPTER ONE

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Judd, C. H. 1927	Experiment with counting of sounds and flashes by children and by adults.	40
	Brownell, W.A. 1928	Apprehension of numbers 3-12 presented in concrete visual form.	1850
	Maclatchy, J.H. 1931	Rote counting and number grouping ability of six-year old children entering first grade.	1242
	Maclatchy, J.H. 1932	Relation between rote and group counting ability and knowledge of addition combinations of six-year old children entering first grade.	708
Minor Research	Douglass, H.R. 1925	Concepts of numbers 1-10 possessed by kindergarten children as tested by dot selection and marble tests.	66
Library Summary	Freeman, F.N. 1912	Recognition of grouped and ungrouped dots by children aged 8-14 and by adults.	
Discussion (Case Study)	Court, S.R.A. 1920	Narration of chronological acquisition of number concepts by child in first five years.	
	Court, S.R.A. 1923	Narration of chronological acquisition of number concepts by child during fifth to eighth years of age.	

CONCLUSIONS

The studies reported in this chapter divide themselves naturally into two types (1) those relating to the nature of the counting process, of grouping, and of number recognition, including the studies by Judd, Brownell, Douglass and Freeman, and (2) those relating to the kind and amount of number knowledge possessed by young children, including the two studies by Maclatchy and the two by Court. This latter type of investigation indicates a natural number interest on the part of children, a considerable body of information resulting from the life experiences of children and argues, as Miss Maclatchy stated, "for child instruction rather than group instruction and for continued use of the method by which he acquired these facts, adapted to school situations of course". The writer would add that, since children learn so well by natural methods, the call for school adaption is not indicated by the study.

Chapter Two

NATURE OF PRIMARY NUMBER - FORMAL OR INFORMAL?

MAJOR RESEARCH

Connor, Wm. L.
A Study of the Kindergarten Achievement Testing in
Arithmetic, First and Second Semester, 1928-1929.
Bulletin #6, Bureau of Educational Research. Board of
Education, Cleveland, Ohio. July 29, 1929.

From observations which were made of the number used by kindergarten and first grade children in class, corridor, playground and street, data were obtained for use as the basis for an achievement test for kindergarten children. This test covers rote counting as far as possible; rational counting to 20 using different sets of objects; comprehension of number group; recognition of number group; thought problems and 12 addition combinations. The test was given in January to 1242 children and in June to 1968 children.

Results of this test show the medians for rote counting to be 28.4 in the January test and 25.9 in the June test. There seem to be differences in the ability to count 20 tacks in a board, 20 marbles or other small objects and 20 balls. About 50% of the children possessed the ability to count 20 objects, 55% and 53% respectively in January and June had mastered the primary addition combinations.

The author suggests the comparison of groups from the kindergarten through ninth grade to test results of formal and incidental teaching in grades one to three.

[illegible]

[illegible]

[illegible] - [illegible]

[illegible]

[illegible]

A Study of [illegible] Achievement Testing in Arithmetic, [illegible]

Bulletin [illegible], Bureau of Educational Research, [illegible] of [illegible]

[illegible]

[illegible]

[illegible] The test was given in [illegible]

[illegible]

[illegible]

[illegible]

Hammond, Walter E.
Securing 100% Accuracy in the Fundamentals of Arithmetic.
Thesis for Master of Education. Boston University, 1929.

The author describes the situation in a New Jersey town where no formal arithmetic was given in grades one and two and where the main drive in grade three was addition with the grocery store used for an informational unit. The Wilson General Survey Test given to 10 third and fourth grade classes in February 1929 was followed by the use of the Wilson Drill Service for Addition. The thesis discusses the use of the drill service and tests. When the test on process steps in addition was given on May 14 no class had 100% but by June 10 all classes except four, in one of which an epidemic of mumps had prevailed, had achieved 100% mastery of addition combinations and process steps.

Wilson, Guy M.
New Standards in Arithmetic: A Controlled Experiment in Supervision.
Journal of Educational Research 22: 351-60. December 1930.

This experiment involved 33 teachers and over 1000 children in the city of Lawrence, Mass., and extended over a period of three years.

The aims of the informational arithmetic in grades one and two were (1) to build number concepts; (2) to extend number experience and (3) to build a basis for understanding and thinking in number situations. Stores with simple situations and right prices were used in both grades. Real games

such as parchesi and ten pins were also used.

A test comprising the addition and subtraction combinations shows the following results in comparison with two cities in which formal work occurred in the first two grades:

L.	100	99	98	97	96	95	94-90	89-80	79-70	69-60	59-50	Q_1	M.	Q_3
Informal 475 children	289	53	54	16	20	13	17	5	4	1	3	100	100	98
B. Formal 174 children	31	33	18	12	4	9	5	9	1	1	1	100	99	98
R. Formal 154 children	48	31	26	9	14	6	12	7	1			100	99	96

The third grade program continued informational arithmetic one day each week, the other four days being devoted to a systematic attack on the number facts. At the end of grade three averages of 97.8% for addition and 95.6 for subtraction were made. 82.7% of the children had perfect scores in subtraction.

The author feels that the data speak for themselves and suggest the possibility of improvement over the present

through the use of informational activities instead of drill in grades one and two.

Woody, Clifford

The Arithmetical Background of Young Children.

Journal of Educational Research 24: 188-201. October 1931.

A test of 205 items was given by teachers in 39 different school systems to children in kindergarten, grades one B and one A, and two B and two A, a total of 2913 children. Each teacher interviewed six children in general.

Kindergarten children knew ¼ of the items, one A children knew ½ and children in two A knew ¾ of the items.

The following table shows the per cent of children in each grade responding to the different phases of counting:

			Kdg.	1B.	1A.	2B.	2A.
Rote	100	1's	26	38	66	76	94
Rote	100	10's	32	46	69	76	93
Rote	30	2's	7	14	34	28	68
Rote	30	3's	3	2	9	11	16
Rote (backward)	20	2's	4	3	12	12	35
Counting 20 circles			71	79	93	98	97
Number of children			94	604	1897	80	238

In telling time the four examples for reading time, viz., 9:00, 1:00, 12:00, and 3:30 were easier than the four examples for placing the minute hand, 7:00, 5:00, 10:00, and 11:45. The boys excelled in this exercise.

In fractions the designated apple cut into halves,

fourths, and thirds got better response than the question "You can cut an apple into how many halves? fourths? thirds?" A large proportion knew that a whole is larger than a half, and about 50% of the children knew that one-half is larger than one-fourth.

Mastery of addition facts shows a wide range of difference as presented in a table.

The author's conclusions are:

1. Children possess much ability before beginning formal instruction.

2. Not only counting and easy addition facts but fractions, United States money, measuring units, and verbal problems are to some extent known by the children.

3. Extended inventory tests must be given first if formal instruction is to be adapted to the needs of the children.

4. The method of attaining the solution should be given consideration.

5. There is a small but persistent tendency for boys to show a slightly greater percentage of ability.

SUMMARY OF CHAPTER TWO

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Connor, Wm. L. 1929	Administration of a test constructed upon observations of number used by children in kindergarten and first grade.	1968
	Hammond, W.E. 1929	Use of Wilson Drill Service for Addition in a third grade in a system providing informal arithmetic in grades one and two.	10 third and fourth grade classes

Wilson, G.M. 1930	Comparisons of results on addition and subtraction facts by children having informal number in grades one and two and those having formal number.	33 teachers and over 1000 pupils
Woody, C. 1931	Results of a 205 item test including rote and rational counting, telling time, fraction meanings, and addition facts.	2913

CONCLUSIONS

In view of the uncertainty, held by many experienced teachers, as to the nature of "formal" and "informal" number, the writer hoped to find a comprehensive discussion of these, as well as an adequate treatment of "incidental" number. Although many articles were encountered which contained descriptions of one or more informal activities, these were in no sense researches. The student interested in informal number would do well to acquaint himself with these illustrations from which he may build his own concepts.

The studies by Connor and by Woody indicate number ability possessed by kindergarten and by first and second grade children and are of greater value as curriculum studies than as method. The Wilson study is also curricular in nature but is the most convincing study of informal work available to the writer. The study by Hammond, while a methods study in a sense, perhaps should be classified as a study evaluating drill material, a field not covered in this thesis.

No doubt much more exploration of informal number

content and methods has been done than had appeared in print. For the benefit of college students whose own primary work has been formal and whose methods courses precede training, the writer hopes that controlled investigation on a large scale may soon take place following the suggestion by Connor that results of "formal" and of "informal" work be tested from kindergarten through ninth grade.

Chapter Three

READING AND WRITING OF NUMBERS

READING

MAJOR RESEARCH

Terry, Paul Washington
How Numerals Are Read.
Supplementary Educational Monograph #18. Department of Education, Chicago University. Chicago, Illinois. 1922.

This monograph treats primarily the reading of numerals as they occur in problems. However two chapters are devoted to the reading of numerals in columns. Chapter 5 reports reading aloud and chapter 10 presents plates showing eye-movements.

Four graduate students read 48 numbers containing from one to seven digits each separated by commas as usual, speaking as they wrote. It was found that: (1) the digits of the numbers were grouped in the process of reading into groups of one, two and three digits; (2) Numbers of each of several digit-lengths were read almost invariably in a main group pattern peculiar to that digit-length; (3) Various numerical patterns were used in pronouncing numbers of the same length; (4) The employment of punctuation with the longer numbers encouraged the use of three-digit groups and conversely discouraged the use of two-digit groups in reading; (5) Main group and numerical-language patterns which were used in the first reading tended to persist for the

most part through a second reading of the same number.

The ten plates contained in chapter 10 show these tendencies; (1) There are two types of pauses (a) reading pauses to recognise identity and (b) guiding pauses to locate initial and final digits; (2) The average total reading time per numeral and the number of pauses per numeral increase gradually from averages of one-digit numerals to averages of seven-digit numerals; (3) With three subjects the average duration of pauses increased with increases in the number of digits read by the pauses; (4) Two different numbers of pauses included the number of pauses used in reading a preponderant number of numerals of each length, the greatest consistency being in one-digit to five-digit numerals; (5) Familiarity rather than regularity reduced the average number of pauses in reading 1000 and 25000; (6) A large number of relatively short pauses resulted in shorter reading time than a relatively few long pauses; (7) The usual number of digits read per pause was one or two, or two or three according to the habits of the subject, one of whom read four digits at a single pause.

MINOR RESEARCH

Lessenger, W.E.

Reading Difficulties in Arithmetical Computation.

Journal of Educational Research 11: 287-91. April, 1925.

111 children given Forms A and B of the Stanford Achievement Test 8½ months apart showed arithmetic loss due to faulty reading on the first test to be 6.1 and on the

second test .7. The mean arithmetic gain due to improvement in reading was 9½ months. A comparison of the gains of 67 poor readers and of 44 good readers shows the gain of the poor readers to be 8½ months in excess of the gain of the good readers.

LIBRARY SUMMARY

Terry, P.W.
The Reading Problem in Arithmetic.
Journal of Educational Psychology 12:365-377. October 1921.

A summary of major research by same author. Describes material used, records results with the reading of numerals occurring in problems and in columns. Two tables present the data.

WRITING NUMBERS

MAJOR RESEARCH

Newland, T. Ernest
A Study of the Specific Illegibilities Found in the Writing of Arabic Numerals.
Journal of Educational Research 21:177-185. March 1930.

The purposes of this research are to determine (1) the tendency of certain Arabic numerals to be misread, (2) the tendency of illegibilities to persist, and (3) the remedial teaching needed to reduce illegibilities.

The materials used were the regular arithmetic papers of 128 children in grades three and four and of 159 children in grades seven, eight, and nine; adult handwriting from 1583 sales slips made by 50 clerks; 852 personal checks

made by 750 persons; 1430 luggage checks; reports of 40 bus drivers; a total of more than 135,000 digits written by 1127 different persons.

Any symbol which required a second glance to see what number was meant was considered as illegible. Reproductions were made of the various kinds of illegibilities with a tally mark for each occurrence of the illegibility.

Three tables show the results of this study.

The conclusions are: (1) that adults write Arabic numerals illegibly about four times out of a hundred or about twice as frequently as do either elementary or junior high school pupils; (2) 6, 5, and 7 in decreasing order are illegibly made by elementary pupils, 5, 7, and 4 by junior high school pupils, and 5, 0, and 2 by adults; (3) 5, 0, and 2 are made illegibly with increasing frequency from elementary to adult level, 4 decreasing in frequency with age; (4) specific kinds of illegibilities do tend to persist in different age groups, e.g. 7, 2, and 5.

The author suggests that 5 be made with one stroke 5 and that the ornamental strokes in 7 and 2 be eliminated.

MINOR RESEARCH

Kallom, A. W.

Times of Writing Each of the Arabic Numerals Determined by the Reaction Time Method.

Journal of Educational Psychology 7: 226. April 1916.

The author as the only subject made each figure twenty-five times recording the time by use of the Bergstrom

Pendulum Chronoscope in thousandths of a second. The experiment was repeated the next day.

The median time in thousandths of a second required for each figure on each day is shown in the following table:

First Day		Second Day	
Number	Median	Number	Median
1	40.1	1	46
*2	224.5	*2	261
2	140.2	2	161.5
3	260	3	262.5
4	241	4	236
5	295	5	298
6	172.5	6	160
*7	259	*7	220
7	114	7	98
8	195.3	8	210
9	170.4	9	184
0	118	0	110.2

*Made with a preliminary stroke.

The author suggests that the knowledge of the time required to write symbols be taken into consideration in the scientific construction of tests.

SUMMARY OF CHAPTER THREE

	Type of Study	Author and Date	Nature of Study	Number of Subjects
Reading Numbers	Major Research	Terry, P.W. 1922	Eye movements and grouping habits used by graduate students in reading numerals in columns.	4
	Minor Research	Lessenger W.E. 1925	Effect of exercises correcting faulty reading habits.	111
	Library Summary	Terry, P.W. 1921	Summary of major research described above.	
Writing Numbers	Major Research	Newland T.Ernest 1930	Study of illegibilities in the writing of Arabic numerals by children and adults.	1127
	Minor Research	Kallom A.W. 1916	Study of time required in the formation of each figure.	1

CONCLUSIONS

The work done by Terry with his graduate students suggests the possibility of determining how numerals are read by school children. Since curriculum studies have suggested reading or informational uses for numerals of several digits, might it not be interesting to investigate the reading and interpretation of these numbers by children reading them? How much do numerical expressions met in general reading, not requiring use or involving judgment, add to the reader's knowledge or appreciation of the article in which they occur?

No investigations relate to corrective measures in the reading of numerals. Is the use of "and" in whole numbers being accepted in teaching circles? The reading of numbers everywhere disregards entirely the omission of "and"

in numbers not containing decimal fractions. Should teachers of arithmetic also disregard it?

The research by Newland gives definite help to the teachers of arithmetic especially those in whose grades children are learning the formation of figures.

Chapter Four

RELATIVE DIFFICULTY OF THE NUMBER COMBINATIONS

MAJOR RESEARCH

Holloway, Harry Vance

An Experimental Study to Determine the Relative Difficulty of the Elementary Number Combinations in Addition and Multiplication.

Doctor's Thesis. University of Pennsylvania. 1914.

Star Gazette Publishing Company, Trenton, N.J.

The author, upon examination of three number primers, finds (1) "no agreement among authors as to the proper number of presentations to be given to the various facts; and (2) no relation between the number of repetitions of a given combination by an author and the degree of difficulty of that combination."

Four investigations, involving children in first, second and third grades, were made by the author. They are as follows:

1. Difficulty as shown in learning the combinations. The coefficient of difficulty was determined by comparing the number of errors made in the preliminary test with the number of errors made in the final test in each combination. The addition and multiplication tests were given to 125 and 180 children respectively.
2. Difficulty as shown by errors made in the combinations was reported for 1056 children in addition and for 1215 in multiplication.
3. Difficulty as shown by the combinations forgotten was

determined by a comparison of the June and September scores made by 558 children.

4. Difficulty as shown by the time required to write the combinations was based on the difference between the time required to write the sums or products and the time required to copy the answers from the back of the test sheet. Test papers for about 350 children were used in this study. The data are fully set forth in 21 graphs and tables.

The most important findings of the author are as follows:

1. "In the process of learning each group of combinations, power is developed whereby subsequent groups are more easily mastered."
2. Doubles in both addition and multiplication and 10's and 11's in multiplication are among the easiest combinations.
3. Combinations in the latter part of the series of both addition and multiplication are more easily learned but more easily forgotten.
4. Difficulty in addition increases with the magnitude of the numbers.
5. A close correlation exists between speed and accuracy in manipulation.

Comment: 45 instead of 100 addition and 78 instead of 100 multiplication combinations were used.

Clapp, Frank L.

The Number Combinations: Their Relative Difficulty and the Frequency of Their Appearance in Textbooks.

Bureau of Education Research Bulletin #2. University of Wisconsin, Madison, Wisconsin, 1924. pp. 126.

This investigation by Clapp is similar to the above study by Holloway in that both authors attempt to determine the difficulty of number combinations and to analyze textbooks for frequency of occurrence of number combinations. Test A involved about 7000 children in grades 3-8. In this test the combinations in addition, subtraction, multiplication and division were presented singly in order to determine the relative extent to which the association between each of the basic combinations and its answer had become automatic. The method of oral presentation was used in this test. The results of this test show learning difficulty. 3500 children in 45 different school systems were given test B in which the primary combinations were presented in examples. Each child worked individually at his own rate in his own way. The results of this test show functional difficulty.

The first and second books (three book series) in each of two sets of texts were analyzed in such detail that 17 tables are required to present the results.

The entire study contains 47 tables.

The results of this study are as follows:

1. Both the learning difficulty and the functional difficulty of the primary number combinations for each of the four processes were determined for each grade and tabulated in order of decreasing difficulty.
2. Tables also show the number of times each combination appeared in each of the texts.
3. Combinations tend to hold the same general position of

relative difficulty through the grades.

4. Zero leads to greater difficulty when it occurs alone than when it occurs in examples.

5. In general, the larger the members of a combination the greater its difficulty although there are many exceptions.

6. Pupils are most nearly automatic in addition followed in order by multiplication, subtraction and division.

7. In solving problems the pupils make the smallest per cent of errors on combinations in multiplication followed by subtraction, addition and division.

8. The coefficients of correlation between the difficulty of combinations when presented singly and when presented in problems are: addition .544, subtraction .390, multiplication .580, and division .022.

9. Members of pairs of combinations are no more nearly equal in difficulty than are other combinations.

10. There are 100 primary combinations each for addition, subtraction and multiplication and 90 for division.

11. The number of actual errors in relation to the number of possible errors is greater for certain phases of each process than for combinations.

12. Negative correlation was found between the difficulty of combinations and their frequency of appearance in the two texts which were examined.

Comment: As pointed out by critics this study neglects the past arithmetic history of the children, discovering combinations least well taught rather than inherent difficulty of number facts.

Knight, Frederic B. and Behrens, Minnie S.
The Learning of the 100 Addition Combinations and the
100 Subtraction Combinations.
Longmans Green and Co., New York. 1928. pp. 82.

The purpose of this investigation was to determine the amount of practice needed to obtain mastery in the 100 addition and the 100 subtraction facts. The subjects in addition were 25 second grade children and in subtraction 15 second grade children. For each child's work on each combination there were kept exact records including all actual responses, the time of all responses in tenths of a second, the place and nature of all instruction, and the occurrence and nature of each error. That the authors consider this a major research in spite of the small number of cases may be seen from their statement, "With only 25 cases in addition, our records run into an unbelievably large number. Only in the next world will the writers repeat this experiment on a group of one or two hundred children."

Data are fully shown in 16 tables.

Tables II and VI show respectively for addition and subtraction combinations (1) the general difficulty rank; (2) the number of practices needed by the 50th percentile pupil for mastery; (3) the number of practices needed by the 80th percentile pupil for mastery; (4) the average time in nearest half seconds for three typical practices in maintenance work; (5) the average errors made in learning.

Forty-five pages are devoted to a comparison of the Knight-Behrens and Clapp rankings summarized as follows:

Clapp finds negative correlation between an addition or

subtraction combination and its reverse. Knight-Behrens find positive correlation.

Clapp finds no rime nor reason in the relative difficulty of addition combinations, each combination being independent of all others. Knight-Behrens find much explanation of difficulty in "size of sums".

Clapp's investigation which reveals upper grade difficulty with the combinations in terms of "frequency of error" can shed no light on the amount of practice needed on each combination. The Knight-Behrens investigation reports amount of practice needed for the 50th and 80th child for both addition and subtraction.

Comment: The failure to check by pre-test facts already known weakens this study. All told Clapp and these authors are in general agreement.

Washburne, Carleton and Vogel, Mabel
Are Any Number Combinations Inherently Difficult?
Journal of Educational Research 17: 235-255. April 1928.

Tests comprising the primary combinations in each process were given to the following number of children in Winnetka: addition to 861 in second grade, subtraction to 535 in second grade, multiplication to 399 in third grade, division to 188 in third grade. Form I of the test contains Clapp's list in order from hardest to easiest, Form II contains the same list in order from easiest to hardest, and Form III contains the same facts alternating hardest and easiest.

The comparison with the Clapp list, one purpose of

the research, together with other findings is given, after presentation of data in 5 tables.

1. When the combinations are arranged in ascending or descending order the same facts are more likely to be missed at the end of the test than at the beginning. When the 13 hardest and 12 easiest facts composed the first quarter of the test this group caused the largest per cent of error. Therefore it seems that position in the test is less dominant than some inherent difficulty of certain combinations.

2. Individual work in Winnetka results in greater accuracy than class work shown by Clapp. Hardest and easiest combinations agree with the Clapp list better than do the intermediate facts. Except in subtraction definite correlation is found between Clapp's findings for Wisconsin and the findings from the three test forms given in Winnetka.

3. Frequency of occurrence in text materials does not seem to have influenced the number of errors made on the various combinations in Winnetka. This may have been due to the system of individual drill on weakness.

4. Addition facts causing difficulty in grades four and five show no relation to facts occasionally missed in grades seven and eight in Winnetka. There is a fairly high correlation between errors in subtraction, multiplication and division in grades four, five, seven and eight.

5. In the hardest quarter of every Winnetka form and of

Clapp's list are found 7 + 9, 9 + 8, 5 + 9 in addition indicating relation between size of addend and relative difficulty; 11-3, 14-6, 16-8, 13-8 showing the presence of 8 as a member in difficult subtraction facts; all zero facts except 0x0 in multiplication suggesting confusion with addition or a difficult concept; and 54 ÷ 9, 9 ÷ 9, 2 ÷ 2, 7 ÷ 7, 5 ÷ 5, 6 ÷ 6, 4 ÷ 4, 3 ÷ 3 in division pointing to size of factors in 54 ÷ 9 and to difficulty when the quotient is due to confusion with subtraction or to difficulty of concept.

6. Those combinations most difficult at the time of teaching, i.e. those missed most frequently at the close of the grade in which taught are not particularly different from the most difficult in the entire study.

7. The Winnetka study agrees with Clapp's findings and results of other studies having comparable data on the difficulty of number facts.

Comment: Does a study of errors in tests of fundamental facts after initial teaching show inherent difficulty or does it indicate those facts which have been least well taught?

Knight points out the fact that a part of the teaching task is to prevent interference and confusion. Is the presence of the zero facts in multiplication and of division facts with the quotient an indication of inherent difficulty or failure to anticipate and prevent interference?

May it not also be that the work tested, occurring early in the course before the children had built up sufficient experience led to the development of confusions as a

result of learning too many things at a time?

Norem, Grant B. and Knight, Frederic B.
The Learning of the One Hundred Multiplication Combinations. Report of the Society's Committee on Arithmetic. Twenty-ninth Yearbook of the National Society for the Study of Education: 551-68. 1930.

This research was conducted with twenty-five third grade pupils by means of an individual drill and test program involving the initial learning of the one hundred primary multiplication combinations. Special features were the absence of drill at home and the counting by an observer of all practices on any fact.

Data are fully treated statistically through the use of 8 tables.

Following are the results of this research:

1. An approximation is given of the amount of practice needed to learn each combination, e.g. 84.20 is the average number of responses given for learning 6x9.
2. One practice a week is often insufficient practice for maintaining combinations during the first two weeks following the initial learning of them but when mastery has been well established it is sufficient for maintenance.
3. The facts are listed in order of difficulty based on composite scores determined from number of learning responses, number of erroneous responses and response time. There is considerable relationship between

size of product and number of responses needed for mastery but no significant support for order of magnitude of two factors as a condition of degree of difficulty. Combinations which are more susceptible to an interchange of multiple are more difficult.

4. The study of errors revealed that (1) about 91% of multiplication errors are correct answers to some other of the one hundred combinations; (2) about 67% of the errors are the correct answers to very closely allied combinations, even when combinations are not taught in tables.

Comment: Meriam, in his discussion of the 29th Yearbook "An Educational Yearbook and Propaganda", School and Society 33: 697-701, says of this study, "The critically minded reader cannot but entertain misgivings as to the reliability of such 'objective data' especially when he notes the small number of pupils in the experiment and the indefiniteness of procedure".

SUMMARY OF MAJOR RESEARCH

Date	Author	Number of Subjects	Processes	Method	Results
1914	Holloway	125	Addition	Tests on 45 primary facts.	Doubles easy. Relation between difficulty and size of addends.
		180	Multiplication	Tests on 78 multiplication facts	Doubles and 10's and 11's easiest.

Date	Author	Number of Subjects	Processes	Method	Results
1924	Clapp	7000	Addition Subtraction Multiplication Division	Tests on all primary facts presented singly	The larger the members of a combination the greater its difficulty. Members of pairs of combinations no more nearly equal than other combinations.
		3500	Addition Subtraction Multiplication Division	Tests on all primary facts in problems	Zero more difficult alone than in problems.
1928	Knight and Behrens	25	Addition	Counting number of practices	Explanation of difficulty in size of sum. Positive correlation between combination and its reverse.
		15	Subtraction	" "	" "
1928	Washburne and Vogel	861	Addition	Tests on all primary facts arranged from the Clapp list	Relation between size of addend and difficulty.
		535	Subtraction	" "	No correlation with Clapp findings.
		399	Multiplication	" "	All zero facts except 0x0 hard.
		188	Division	" "	Size of members related to difficulty. Quotient cause of difficulty.
1930	Norem and Knight	25	Multiplication	Counting number of practices	Considerable relationship between size of product and number of responses. No significant support for order of magnitude as condition of degree of difficulty. Large per cent of errors correct response to some other combination.

MINOR RESEARCH

Smith, J. H.

Arithmetical Combinations.

Elementary School Journal 21: 762-770. June 1921.

This study is based on the time response for individuals on the 81 primary addition combinations (zeros omitted), the 100 subtraction combinations, the 100 multiplication combinations and 80 division combinations (division by 1 omitted).

The number of subjects varied, - 86 training school children and one methods class being used in addition, 26 older students in subtraction, 42 pupils in multiplication and 18 pupils in division.

Results are given for each process:

1. Addition: Little difference is found in response time between a combination and its reverse. Easy combinations have sums not exceeding 10. Doubles occur in the easy group or in the first half of the medium group.
2. Subtraction: With three exceptions difficult minuends are 10 or more. 11-9 and 11-3 are causes of many errors.
3. Multiplication: Although zero combinations were answered incorrectly by several children the author feels that they may not be inherently difficult. Difficult facts have 6, 7, 8, and 9 as multipliers.
4. Division: Zero was given as responses to 4 ÷ 4, 5 ÷ 5, 8 ÷ 8, and 9 ÷ 9.

Comment: Failure to include the entire 100 addition combina-

tions and the 90 division combinations makes this study incomplete. There is not adequate description of the method used.

Batson, Wm. H. and Combellick, Olin E.
Relative Difficulty of Number Combinations in Addition and Multiplication
Journal of Educational Psychology 16: 467-481. October 1925.

Mental and physical response time were noted by means of kymographic records for each of the 100 primary addition and the 100 primary multiplication combinations. The 83 subjects participating in this study included 1 Ph.D., 22 B.A. or B.S., 8 college seniors, 11 juniors, 19 sophomores, 12 freshmen, 9 high school pupils and 1 sixth grader. The combinations were arranged on series of cards the answers being written in the openings below each combination.

The findings are clearly and fully presented in 6 tables.

Errors, mental time median, physical time median and sum of mental and physical medians are shown for each fact.

Results of the error study show sums involving 5 and 9 occurring the same number of times and each causing 15.45% of all errors. These addends were followed by 4 which causes 10.16% of all errors. In multiplication 0 leads as a cause of error followed by 9, 7 and 8. The authors conclude from tables showing the relative difficulty of combinations in order of decreasing time that difficulties do not lie entirely with the numbers of larger numerical value. The range of mental difficulty of the combinations is found to be less than the range of physical difficulty.

Comment: The weakness of this study is recognised by the authors who state that their findings may be due to the native difficulty of the combinations or to the poor distribution of practice.

Fowlkes, John Guy
A Report of a Controlled Study of the Learning of Multiplication by Third Grade Children.
Journal of Educational Research 15:181-189. March 1927.
See Chapter 7 on Multiplication.

LIBRARY SUMMARIES

Bush, Maybelle G.
The Fundamental Number Facts.
School and Society 6:265-268. September 1, 1917.
This article contains a review of the study made by Holloway.

Clapp, Frank L.
The Number Combinations: Their Relative Difficulty and the Frequency of their Appearance in Textbooks .
Bureau of Education Research Bulletin #1. University of Wisconsin, Madison, Wisconsin. 1924.

Bulletin #1 contains a summary of the problem, procedure and results of the investigation reported in Bulletin #2. tables are included in this summary.

Buckingham, B. R.
The Difficulty of the Number Combinations.
Journal of Educational Research 12:387-391. December 1925.

This article briefly summarizes the studies of Holloway, Counts, Smith, Clapp, and Batson and Combellick. The author sees a value in the results of these investigations both for the text-book writer and the teacher, making easier the provision for better balanced drill.

Ruch, G. M.
Relative Difficulty of the One Hundred Multiplication Facts with Special Reference to Textbook Construction.
Elementary School Journal 32: 369-377. January 1932.

This article presents a comparison of the studies of relative difficulty of multiplication facts made by Norem and Knight, Clapp and Fowlkes.

These limitations of the Clapp study are pointed out by the author:

1. The previous number experiences of the pupils tested were largely unknown quantities.
2. The text-books studied by the experimental group were not modern and had not been constructed with the idea of calculated number experiences.
3. The difficulties, as ascertained, cannot be viewed as learning difficulties but must be regarded as semi-persistent difficulties under haphazard instruction.

The studies by Fowlkes and by Norem and Knight are similar in being studies of initial learning with amounts of practice known and controlled. The author's criticism of these studies is that the number of children participating was not as large as could be wished.

The degree of agreement in difficulty ranking seen

in these correlations: Norem and Knight with Clapp 0.34; Norem and Knight with Fowlkes 0.83; Fowlkes with Clapp 0.42. The author concludes that the "difficulties which the one hundred multiplication combinations present during the stage of initial learning are quite different from the final difficulties at the end of instruction". The greatest disagreement is in the relative difficulty of the zero facts. With the exception of zero times zero Clapp finds all zero facts in the most difficult quarter. Further study of the zero combinations appearing in each quarter of difficulty in the different studies leads to two conclusions (1) that "initial learning is not to be confused with difficulties found to persist after some years of instruction" and (2) " the so-called zero difficulty is principally an artifact of a policy of neglect in older text-books."

The author points to relation between order of difficulty and size of product presenting the Clapp correlation 0.61 and the Norem and Knight correlation 0.86. He holds that the high correlation between size of product and order of difficulty, found in both the Fowlkes and the Norem and Knight studies, is a valid answer if extended to a larger sampling of pupils, teachers and methods.

Four tables present the author's findings. Summarizing, he states these conclusions:

1. Learning difficulties are very different from final difficulties.

2. There is a rough agreement of the order of difficulties in learning in the Fowlkes and Norem and Knight studies.

Boston University
School of Education
Library

(The correlation of 0.83 is unquestionably diluted by the small numbers employed.)

3. There is considerable correlation of difficulty with size of product.

4 . Clapp's ranking of difficulty agrees somewhat with distribution of practice in texts studies by his pupils, but agreements with other studies are slight.

5. Some such sensible distribution of practice as the following is probably as close to the truth as we can come at present

Quadrant I	Quadrant II
20%	25%
Quadrant III	Quadrant IV
25%	30%

This assumes that difficulty is correlated with size of product in multiplication combinations.

6. The so-called zero difficulties arise from neglect; they disappear under controlled instruction.

SUMMARY OF CHAPTER FOUR

Major Research	Minor Research	Library Summary	Discussion
Holloway 1914	Smith 1921	Bush 1917	
Clapp 1924	Batson and Combellick 1925	Clapp 1924	
Washburne and Vogel 1926		Buckingham 1925	
Norem and Knight 1928		Ruch 1932	
Knight and Behrens 1928			

CONCLUSIONS

The initial investigation in this field by Holloway, although limited to 45 addition and 78 multiplication facts, opened up an interesting problem which was later studied more fully by Clapp. The Clapp study in turn led to further studies by Washburne and Vogel and by Knight and Behrens. The studies by Clapp and by Washburne and Vogel are error studies made after the initial teaching of the number combinations. The investigations by Knight and Behrens and by Norem and Knight are based on the number of repetitions of each number combination needed for mastery, but without pre-test.

There are evidences in several studies that the cause of considerable error is due to interference between answers to combinations, e.g. 0 in multiplication confused with 0 in addition. It would be helpful to one who is interested in methods of teaching the combinations to know what interferences are likely to occur and how to prevent such occurrence.

Chapter Five

ADDITION

MAJOR RESEARCH

Buckingham, B. R.
Teaching Addition Facts Together or Separately.
Educational Research Bulletin (Ohio State University) 6: 228-229 & 240-242. May 25, 1927.

This article describes the teaching in seven centers of 38 primary addition and their related subtraction facts. The children - the number not noted - were grouped and paired according to their results on the Pressey Primary Classification. The reverses of the facts were taught with the facts whether addition and subtraction facts were taught together or separately.

Two tables present the results for all centers based on a final test of the 38 facts.

Results in all centers but one indicate that the differences are in favor of teaching the facts together.

The author recognises that the teaching material should include all addition and subtraction facts and that no testing for retention was done because the year was so far advanced.

What grade? —

Buckingham, B. R.
Upward Versus Downward Addition.
Journal of Educational Research 16: 315-322. December 1927.

A review of the 1925 discussion by the same author is included in this article.

Each class participating in this controlled experiment in the seven centers was divided into two sections on the basis of results on a test on 50 primary and 50 upper decade facts. Second or third grades were used depending on the place of the initial teaching of column addition. Twenty minutes daily were devoted to this work, all drill and review being in class with no home work. The experiment ended when the pupils were reasonably proficient in adding short columns of one-place numbers. The final test contained 50 examples, the two forms using the same facts but in reverse order, e.g.

A.	B.
5	6
4	3
3	4
6	5

Two tables give the results at Center I and a summary of the results at the seven centers. Downward addition is favored in all but one Center.

The author recognises the limitations and questions if the verdict would have favored downward addition if the examination had been more exacting. In the author's opinion however there are two reasons to expect that downward addition is more advantageous, viz. the eye and hand are not far from the place where the sum is to be written in downward addition and (2) more errors are made in carrying because more movements are required in adding upward.

Comment: The differences are so small that it is doubtful if a case has been proven.

Maclatchy, Josephine

Addition in the First Grade.

Educational Research Bulletin (Ohio State University) 9: 391-393 & 406. October 8, 1930.

This article contains a more detailed account of the addition part of an extended survey made by Buckingham and Maclatchy and reported in the Twenty-ninth Yearbook of the Society for the Study of Education. An addition test for children entering school was made up of ten combinations, viz. 5 + 1, 7 + 1, 1 + 9 , 4 + 4, 1 + 6, 5 + 2, 8 + 2, 4 + 5, 5 + 3, 3 + 5. These were chosen from the Knight-Behrens list beginning with the second and taking facts at equal intervals after eliminating zeros and sums over 10. 1226 children in ten cities of Ohio, 1123 from Cincinnati and 130 from rural schools all of whom were from six to six and one-half years of age were given this test during the first two weeks of school in September 1928.

Two tables show the percentages of familiarity with the ten facts and the combinations ranked by percentages.

The author finds that country children are less familiar than city children with all except 5 + 1. All three groups agree in finding 5 + 1, 7 + 1, 1 + 9 easiest and 3 + 5, 4 + 5 the hardest. 11% knew no sum, 7% knew all ten facts. The median for those knowing some but not all was 4.7.

Miller, Florence May

Ability of Third Grade Children to Comprehend a Mastery Plan for Addition.

Master's Thesis, Graduate School Boston University, 1930.

The work done by a third grade of 38 children is reported in detail in this thesis. These children who had followed a program of required arithmetic in first and second grades showed in their review work evidence of counting habits.

In the use of the Wilson Drill Service for Addition these points were observed: (1) the presentation of the work as outlined; (2) the distribution of drill according to relative difficulty of facts and processes; (3) the recording of individual errors and particular needs; (4) the constant checking of own work by the pupil; (5) the observation of the laws of memory.

Tests 3A, 3B, 3C, 3D, and 3P were used containing respectively easy primary facts, harder primary facts, upper decade facts needed for column addition, upper decade facts needed for carrying in multiplication, and process steps in addition. In April these tests were all given for a second time.

The following table gives a comparison of the second test with the first:

Test	Range	Median	S.D.	A.D.	Date
3A 1	80-100	100	3.73	1.55	November 1
3A 2	100	100	0	0	April 1
3B 1	87-100	99	3.17	2.00	November 8
3B 2	99-100	100	.16	.026	April 2
3C 1	51-99	96	11.21	5.31	January 4
3C 2	97-100	100	.842	.394	April 3
3D 1	54-100	97	12.87	6.105	January 18
3D 2	92-100	100	1.56	.552	April 4
3P 1	70-100	87	8.15	6.44	February 1
3P 2	90-100	99	2.44	1.21	April 5

No conclusions, as such, are stated by the author except to say that the comprehension and execution and especially the self direction of the children were a revelation to her.

MINOR RESEARCH

Cole Lawrence W.

Adding Upward and Downward.

Journal of Educational Psychology 3: 83-94. February 1912.

Thirty persons selected at random added twenty columns of forty numbers each, half of them adding upward and half adding downward.

The author finds that the amount of error was less for adding downward, and that downward addition was slower.

No tables are shown.

Comment: The group participating was too small, groups were not equated and there was no control of factors.

Phelps, C. L.

A Study of Errors in Tests of Adding Ability.

Elementary School Teacher 14: 29-39. September 1913

Twenty-five arrangements of 45 addition combinations were given to 270 eighth-grade children. These were given in the form of one-minute tests, five each day for five days being administered.

Data are fully presented in five tables.

Tests results show that 7 + 9, 5 + 8, 6 + 9, 3 + 9, 5 + 9, 8 + 9, 4 + 8, 7 + 8, 5 + 6, and 5 + 7 led to the greatest per cent of error. Except for a few zero errors and a few erratic type errors the number of errors and the number of children

making them increased with the increase in the size of the combination. A large percentage of children made mistakes on each of the larger combinations. The occurrence of type errors is noted, 1 + 7 = 9, 8 + 8 = 18, 1 + 4 = 6 occurring over 100 times. No correlation was found between speed and accuracy.

Hahn, H. H. and Thorndike, E. L.
Some Results of Practice in Addition Under School Conditions.
Journal of Educational Psychology 5: 65-84. February 1914.

One hundred ninety two children in grades four, five, six and seven were given practice in adding columns of 10 one-place numbers. The ninety minutes of practice between the 15 minute initial test and the 15 minute final test were divided into periods of 5, 7½, 10, 11¼, 15, 20, and 22½ minutes for the different groups participating. The control groups took the initial and final tests. There was no control of home work.

Seven tables and 12 figures show the results of the experiment.

The conclusions of the authors are (1) that pupils will gain 1% a minute in the amount done correctly from an hour of distributed adding; (2) that pupils who had special practice gained 80% in the number of examples correct; (3) that short periods are not more favorable than longer periods, the evidence not being secure but in favor of longer periods; (4) that there was an enormous difference in initial ability and amount of gain and (5) that the effect making them increased with the increase in the size of the combinations. A large percentage of children made mistakes in each of the larger combinations. The occurrence of type errors is noted, 1 + 2 = 9, 4 + 5 = 10, 1 + 4 = 6 occurring over 100 times. No correlation was found between speed and accuracy.

Mead, C. D., and [illegible]
Some Results of Practice in Addition under School Conditions. Journal of Educational Psychology 21: 55-62. February 1930.

The selected ninety-two children in grades four, five, six and seven were given practice in adding columns of 10 one-place numbers. The amount, minutes of practice between the 1-minute initial test and the 12 minute final test were divided into periods of 5, 7½, 10, 11, 15, 20, and 30 minutes for the different groups participating. The control groups took the initial and final tests. There was no control at home work.

These tables and 12 figures show the results of the experiment.

The conclusions of the authors are (1) that pupils will gain 1.5 minute in the highest score correctly from an hour of distributed drill; (2) that pupils who had greatest potential gained in the number of examples correct; (3) that short periods are not more favorable than longer periods, the evidence not being certain but in favor of longer periods; (4) that there are no constant differences in initial abilities ... amount of gain and (5) that the correct of practice was to increase rather than decrease individual differences.

Chapman, J. Crosby
A Study of Initial Spurt in the Case of Addition.
Journal of Educational Psychology 6 : 419-426. September 1915.

This article describes an experiment which involved 46 college men who added ten one-place numbers during 10 ten-minute periods, these periods being on five days with two hours between periods.

Greater efficiency was found to exist during the earlier periods, as follows:

Minutes	Score
0-2	100
2-4	92.5
4-6	89.4
6-8	88.0
8-10	87.1

Bradford, G. G.
An Experiment in Adding.
School Science and Mathematics 16: 328-336. April 1916.

This article reports an experiment in which two subjects practiced adding twenty columns of ten digits each over a period of 139 days. There was variation in the practice, some adding being vertical, some horizontal and both upward and downward addition being used.

The practice curves took a typical form. There was

marked retardation in the horizontal form. One observer made little gain in accuracy because of lack of interest. Concentration was difficult in case of fatigue, indisposition or distraction. A table gives the time in minutes on each series for each day for each observer.

Comment: Of interest psychologically but of no particular value from the viewpoint of arithmetic teaching.

Wilson, Earle E.
Correlation Between the Oral and Written Work of Pupils in the Fundamentals of Addition.
School and Society 5: 300. March 10, 1917.

Forty-two pupils in grade three B were given 100 examples made up of 45 addition combinations. The oral test was given individually, the written test following the next day. The pupils were timed on their responses.

The mean for the written work was 98 as was the median. The mean for the oral work was 92 and the median was 96. There was a fair degree of correlation. The author concludes that children do better written work but it takes about 1/3 longer.

Reed, H. B.
Fatigue and Work Curve from a Ten-hour Day in Addition.
Journal of Educational Psychology 15: 389-392. September 1924.

Eight subjects worked from 7.30 A.M. to 5.30 P.M. with 35 minutes for lunch adding examples with five two-place addends. Increase in right minus wrong from the ten-minute period before rest to the ten-minute period after rest was

chosen as the fatigue measurement.

The fatigue factor measured as above is found to be 21.2%. No evidence is found of initial spurt, end spurt nor adaptation.

Comment: Of interest psychologically but not from arithmetic viewpoint.

Ballenger, H. L.

Overcoming Some Addition Difficulties.

Journal of Educational Research 13: 111-117. February 1926.

Children in grades four, five and six were given a series of eight tests in addition increasing in difficulty from primary facts to columns of nine addends each a three-place number. From the test results classes were divided so that one section contained those children who could add nine figures as well as they could add four or five figures. Children who had difficulty in holding their attention upon a column of more than five figures constituted the second section. The first section was instructed to write the sum of each column at the right until the column was checked and to write the figure to be carried at the top of the next column. The second section divided the columns, added each half, finally adding the two totals for a final result.

Since the purpose of this plan was "to provide for the needs of the individuals who had difficulties of a similar character", the findings are not stated in terms of relative effectiveness of the two methods but in gain in accuracy as represented by the grade medians. Grade medians rose from

the following per cents at the opening of school: four B, 38; four A, 47; five B, 62; five A, 46; six B, 38; six A, 50 to 100% in the January test.

Comment: The type of practice material used goes beyond social needs. This study shows that adapted teaching gives results. If 100% is possible here it is easily so on examples limited to social needs.

LIBRARY SUMMARIES

Waterson, C. E. and Unruh, H. B.
A Critical Study of Errors in Addition Combinations.
Bulletin of Education. Bureau of School Service and Research
Vol. 2 #3: 19-21. February 1929. University of Kansas,
Lawrence, Kansas.

This article contains a review by Waterson of a study by Unruh.

Unruh's study consisted of four separate testings of 161 children in addition combinations. Each test contained 110 examples with one- place addends and 55 examples with two-place addends. Waterson states Unruh's problems to be "(1) What errors occur most frequently? (2) Errors are how persistent? Within each grade? Within three grades? (3) What is the relation of errors to the amount of practice? (4) What is the distribution of errors with respect to the mental ages of the pupils?" Waterson finds that Unruh determined the frequency of error but ignored questions two and three. Findings in this study show that the per cent of zero errors was greater in grade four than any other grade, 1 + 0 having

the highest per cent of error. 1 + 6 and 3 + 3 were first and second in difficulty in grades five and six. 1431 errors were made by 161 pupils. 11.53% of the errors were made by pupils above Q_3 of the total distribution, 12.57% were between Q_2 and Q_3; 30.46% were between Q_1 and Q_2; and 45.42% were below Q_1. No program of teaching, drill or testing procedure is proposed.

Board of Education of the City of New York.
Educational Measurements for the Class Teacher.
Inventory of Errors and Remedial Suggestions - Addition and Subtraction. #11. November 1929.

This bulletin contains a summary of the sections pertaining to addition and subtraction of "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.

DISCUSSIONS

Buckingham, B. R.
Adding Up or Down: A Discussion.
Journal of Educational Research 12: 251-261. November 1925.

The author reports the preference of 493 summer school students most of whom had had teaching experience, as to adding upward or downward. 303 preferred adding upward and 190 preferred adding downward. 468 found no difficulty in adding upward and 449 found no difficulty in adding downward. 104 or 54.7% thought downward adding more reasonable. When asked to read $\begin{array}{c}5\\ \underline{3}\end{array}$ 75% read down.

The author's conclusions are (1) that the preference for adding up is not as strong as supposed, (2) that the preference for upward addition is not incompatible with reversal on occasion.

Five reasons are given by the author for downward addition, viz., (1) the eye-movements are as in ordinary reading, (2) the figures in the example are written down, (3) the sum is arrived at at the place in which it is to be written, (4) the eye-movements are less complex, and (5) this method lends itself to analytical devices.

Douglass, A. H.
Number Combinations as Specific Acts of Learning.
Primary Education-Popular Educator 44:832. June 1927.

Describes the teaching of 45 addition facts. Points out the counting fault but gives it no emphasis.

Comment: Not especially helpful.

Manual, H. T.
Adding Up or Down: Another Consideration.
Journal of Educational Research 18:297-298. April 1928.

In a news item exception is taken to Buckingham's implication that we should teach either upward or downward addition. The author believes that both must be taught for checking purposes. He says that "the significant task is that of finding specific difficulties encountered in each - an analytic study that will furnish a basis for an improved technique in teaching each".

Kramer, Sister Mary Immaculata

Permanence of Improvement and the Distribution of Learning in Addition and Subtraction.

Ph. D. Catholic University of America. Washington, D.C. 1931.

See chapter on Drill.

SUMMARY OF CHAPTER FIVE

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Buckingham B.R. 1927	Investigation of teaching addition and subtraction facts together and separately.	Seven school centers - number of children not noted
	Buckingham 1927	Upward and downward addition.	Seven school centers - number children not noted
	Maclatchy J. 1930	Knowledge of ten addition facts possessed by children entering first grade.	2479
	Miller F.M. 1930	Third grade addition on mastery plan basis.	38
Minor Research	Cole, L.W. 1912	Upward and downward addition.	30
	Phelps, C.L. 1913	Errors in Addition.	270
	Hahn, H.H. and Thorndike E.L. 1914	Effect of practice in addition and length of practice periods.	192
	Chapman J.C. 1915	Initial spurt in practice in addition.	46
	Bradford G.G. 1916	Practice curves in addition.	2
	Wilson E.V. 1917	Correlation between oral and written work.	42

	Reed, H.B. 1924	Fatigue and work curve.	8
	Ballenger H.L. 1926	Drill in addition adapted to children's needs.	Children in grades four, five and six.
Library Summaries	Waterson C.E. and Unruh, H.B. 1929	Review by Waterson of study by Unruh.	161
	Board of Education of City of New York 1929	Summary of sections pertaining to addition and subtraction from "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.	
Discussions	Buckingham B.R. 1925	Upward and downward addition	493
	Douglass A.H. 1927	General discussion.	
	Manual H.T. 1928	Upward and downward addition.	
	Kramer Sister Mary 1931	Permanence of improvement and distribution of learning in addition and subtraction.	

CONCLUSIONS

The problem receiving most consideration, that of adding upward or downward, has not been solved in a convincing manner to date. Cole, whose study involved few people working under uncontrolled conditions, has contributed less than Buckingham, whose early study based upon the opinions of summer school students was followed by a controlled study in which he feels that the examination was not exacting enough to discover the whole truth. Since in both of his articles Buckingham notes reasons for expecting downward addition to be more advantageous than upward addition, his

own opinion on this point is clear. It seems to me that Manual has presented the case more sensibly in suggesting that both upward and downward addition must be taught for checking purposes and that the important factor is an improved technique in teaching each.

Buckingham appreciates the limitations of his study relating to the teaching of addition and subtraction facts together or separately. Perhaps since both sets of facts must be taught it makes little difference whether they are taught together or separately but it would be interesting to conduct a study in which all of the facts were taught and in which testing for retention was done, thus eliminating the weaknesses of Buckingham's study.

The testing, done by Miss Maclatchy, to determine knowledge of addition facts possessed by children entering grade one is suggestive of further inventory testing. Would it be possible to test gradually the 100 primary facts instead of limiting the number of facts tested?

The studies relating to the actual teaching of addiion include Miss Miller's detailed report of the use of the Wilson Drill Service and Ballenger's remedial work on column addition. Counting instead of adding is recognised as a difficulty in addition and is discussed with teachers in University classes but is mentioned but little in published materials. Would not discussion of the best known ways of preventing counting aid at present? Is there not a place for a study comparing the results from activities used to give meaning and a drill service used to give mastery with the objective

presentation of number facts suggested in text-books and still commonly used by teachers?

Error studies in addition are closely related to studies of the relative difficulty of number facts. The studies made by Phelps and by Unruh may be compared with the studies in Chapter Four.

For remedial work in addition the summary by the Board of Education of New York City of the study by Lazar is most helpful.

The comparison by Earle E. Wilson of oral and written work in addition suggests the possibility of determining the social situations in which computation without paper and pencil is necessary or desirable. On the basis of this knowledge experiments would determine further comparisons and desirable techniques in oral and written work.

Addition has been a fertile field for psychological experimentation. The studies of Hahn and Thorndike, Chapman, Bradford, and Reed are of this nature. The computation used is beyond social usage and is only incidental to the use which is made of it.

Chapter Six

SUBTRACTION

FOREWORD

Uniformity in the names of subtraction methods is so lacking as to necessitate some attempt at standardizing terminology. The simplest and most descriptive names are found in "Teaching Arithmetic in the Intermediate Grades" by R.L.Morton. Using the example $\begin{array}{r}9123\\ \underline{4714}\end{array}$ the four methods are illustrated as follows:

Take-away-borrowing.

Think 4 from 13, 1 from 1, 7 from 11, 4 from 8. Other names for this method are decomposition and first Italian.

Take-away-carrying.

Think 4 from 13, 2 from 2, 7 from 11, 5 from 9. Other names for this method are equal additions and second Italian.

Addition-borrowing.

Think 4 and 9 are 13, 1 and 0 are 1, 7 and 4 are 11, 4 and 4 are 8.

Addition-carrying.

Think 4 and 9 are 13, 2 and 0 are 2, 7 and 4 are 11, 5 and 4 are 9.

Although both 3 and 4 are called Austrian, 4 is more frequently called by this name.

Following each article a statement will tell the

Chapter IX

SUBTRACTION

METHODS

Uniformity in the usage of subtraction methods is so lacking as to necessitate some attempt at simplifying terminology. The simplest and most descriptive names are found in "Teaching Arithmetic in the Intermediate Grades" by R.L.Morton. Using the example 9152 / 4614 the four methods are illustrated as follows:

Take-away-borrowing.

Think 4 from 12, 1 from 4, 6 from 11, 4 from 8.

Other names for this method are decomposition and first Italian.

Take-away-carrying.

Think, 4 from 12, 2 from 5, 6 from 11, 5 from 9.

Other names for this method are equal additions and second Italian.

Adding-borrowing.

Think 4 and 8 are 12, 1 and 3 are 4, 6 and 5 are 11, 4 and 4 are 8.

Adding-carrying.

Think 4 and 8 are 12, 2 and 3 are 5, 6 and 5 are 11, 5 and 4 are 9.

Although both 3 and 4 are called additive, 4 is more frequently called the Austrian method.

Following each method a statement will tell the use of names by the author of the article.

MAJOR RESEARCH

Taylor, Joseph S.
Subtraction by the Addition Process.
Elementary School Journal 20:203-207. November 1919.

Four questions raised as to subtraction are: What is the effect of additive subtraction? Is it better? Why? Do children after six years use this method? The fourth question is answered by an investigation of the subtraction method in use by 11,368 children in grades four, five, and six in New York, after the additive method had been official in that city for six years.

21.8% of the children were found to be using the first Italian, 40.6% the second Italian, and 37.6% the Austrian method. The proportion of children using the official method declined from 52.7% in the fourth grade to 21.2% in the sixth grade.

The author asks why the additive method should be used and refutes the stock argument that only one set of primary facts is necessary. He accounts for the use by the children of methods other than the official through the failure of principals and teachers to teach that method. This in turn is due to the difficulty of constructing problems and of rationalizing this process.

Comment: First and second Italian methods are take-away-borrowing and take-away-carrying respectively. Austrian is in this case addition-carrying.

Buckingham, B. R.

Teaching Addition and Subtraction Facts Together or Separately. Educational Research Bulletin (Ohio State University) 6:228-229 & 240-242. May 25, 1927.

See Chapter Five on Addition.

Buckingham, B. R.

The Additive Versus the Take-away Method of Teaching the Subtraction Facts.

Educational Research Bulletin (Ohio State University) 6:265-269. September 28, 1927.

This experiment involved 110 pairs of children grouped as to scores on the Pressey Primary Classification Test. Controlled factors were course of study, amount of time, absence of home work, and same final test.

Table 1 shows paired results at Center I and Table 2 shows results in all centers. The take-away group had 13.5 more combinations correct on the final test with a probable error of .64. In all but one of the seven centers the take-away was favored by 7.2 to 13.5.

The author recognises that this study deals only with immediate recall and tells nothing of the efficiency of either method in regard to retention or when the facts occur in examples.

Osburn, W. J.

How Shall We Subtract?

Journal of Educational Research 16:237-246. November 1927.

This article contains discussion, report of research, and summaries of studies.

The discussion concerns an analysis of methods. Twelve methods are obtained by classifying the additive, take-away, and complementary as to whether borrowing or carrying occurs and further classifying as to whether the subtraction is upward or downward.

The investigations of Ballard, McClelland, Winch, Mead and Sears, Taylor, Beatty, and Stone are summarized.

The research reported by this author involved 1414 pupils in grades four, five, and six, to whom a test was given, the methods were explained, and the question was asked as to the method used.

Two tables show the test results.

The additive decomposition produced the worst results, the mean number of errors being 2.8. The take-away decomposition was almost as bad. The equal-additions methods resulted in fewer errors, the least number being a mean of 1.6 in the take-away equal-additions. Inferior pupils did more poorly with the borrowing methods than they did with the carrying methods. The author feels certain of the superiority of equal-additions over decomposition and has but little doubt of the superiority of take-away over additive methods.

Comment: In this article take-away-decomposition and take-away-equal additions are take-away-borrowing and take-away-carrying respectively. Additive-decomposition and additive

equal additions are addition-borrowing and addition-carrying respectively.

Comment: The differences found are small. Perfect scores are not obtained by any method and perfect scores only are acceptable.

Division of Tests and Measurements, St. Louis Public Schools.
Methods of Subtraction.
Public School Messenger 26:28-32. September 1, 1928.

This investigation, concerning the subtraction method prescribed in the larger cities of the United States, finds that no method is prescribed in Boston, Buffalo, Chicago, Cincinnati, Jersey City, Los Angeles, and Newark. The borrowing method is prescribed in Detroit, Minneapolis, Philadelphia, Pittsburg, Rochester, and Washington. The additive or Austrian method is prescribed in Baltimore, Indianapolis, Milwaukee, New York and Seattle. A second investigation, to determine the methods being used in St. Louis, involved 1065 children chosen at random from the ninth grade and the upper quarter of the eighth grade. A three minute test was given and the method analyzed on the basis of questions.

One table shows the median attempted, the median correct, the per cent correct, and the correlation between the number attempted and the number correct.

Mixed methods were used by 29 with as good results, the Austrian method was used by 60, the take-away-equal additions by 320, and the borrowing by 616.

The author finds that those who used the take-away-equal additions method attempted more and solved more correctly and that there was a higher correlation between speed and

accuracy. The author feels that there is no need for decomposition on the basis of rationalizing with bundles of splints. He says that it is a matter of doubt which method should be prescribed if any.

Comment: In this article borrowing and take-away-equal additions are respectively take-away-borrowing and take-away-carrying. Additive or Austrian is the addition-carrying method.

MINOR RESEARCH

Mead, Cyrus and Sears, Isabel
Additive Subtraction and Multiplicative Division Tested.
Journal of Educational Psychology 7:261-270. May 1916.

The problem in this study was what children do with additive subtraction and multiplicative division when tested over a period of time. Two second grades were used for subtraction and two third grades for division. Grouping for the experiment was on the basis of German or English being studied. A pre-test of the 45 addition combinations was given by one teacher. Tests were given in March, April, May, and June.

The take-away class was 4.5 points above the additive in ability in separate combinations and .7 above in longer examples. The papers of the additive class showed confusion, the tendency being to add instead of subtracting. In division the multiplicative class scored 4.3 above the "into" class in combinations. The "into" class scored .7 above the multiplicative class in longer examples. Final tests show little

relation btween ability in separate tables and ability to work examples.

Comment: Inconclusive, all told. Drill begun too early may account for results. Although examples are mentioned no illustrations show whether borrowing or carrying takes place.

Beatty, Willard W.

The Additive Versus the Borrowing Method of Subtraction.

Elementary School Journal 21:198-200. November 1920.

Following the investigation of Taylor, whose work is summarized and criticized in this article, the author tested 83 children who had been taught the additive method in the training school of the San Francisco Normal School.

He finds that 51.8% of the subjects tested in from four to five years had abandoned the additive and adopted the borrowing method. 78% of the children tested in the San Francisco schools used the borrowing method. Those using the additive method were more accurate but slower.

The author feels that the prescription of one method is not justified by the evidence. He raises this question: If the borrowers were more rapid with an adopted method what could they have done if taught that method?

Comment: The same three types of subtraction are used here as in the study by Taylor. Borrowing is probably take-away-borrowing and additive is addition carrying.

relation between ability in approximate taking and ability to work examples.

Comment: Inconclusive, all told. Drill helps not only any account from results. Although examples are mentioned as illustrations when whether borrowing or carrying takes place.

Beatty, Willard W.

The additive versus the borrowing method of subtraction. Elementary School Journal 21:198-200. November 1920.

Following the investigation of Taylor, whose work is summarized and criticized in this article, the author tested 81 children who had been taught the additive method in the training school of the San Francisco Normal School.

The fifth Grade, 45 of the subjects started in from four to five years and abandoned the additive and adopted the borrowing method. 70% of the children tested in the San Francisco schools teach the borrowing method. Those using the additive method were more accurate but slower.

The author feels that the substitution of one method is not justified by the evidence. He raises this question: If the borrowers were more rapid with an adopted method would they nevertheless be taught that method?

Comment: The most basic types of subtraction are used here as in the study by Taylor. Interesting as providing data for comparing borrowing and additive in addition exercises.

Johnson, J. T.
The Merits of Different Methods of Subtraction.
Journal of Educational Research 10:279-290. November 1924.

A test containing seventeen examples from the Courtis practice pads #33 was given to 277 students in Chicago Normal College.

Four methods were found in use; decomposition being used by 220, equal additions by 23, addition-borrowing by 8, and addition-carrying by 13. The author concludes that the decomposition method is not the best. He finds reason for thinking that a method which increases the subtrahend is better than a borrowing method. He suggests further investigation to see which variation of the carrying method is preferable.

These findings are challenged by Ruch, Knight, and Lutes in the Journal of Educational Research 11:154-155, February 1925, on the basis of meagre sampling, examples too easy, error in computation, and selective tendencies of those using the minority method. This in turn is answered by Johnson in the Journal of Educational Research 12:80-83, June 1925, and followed on pages 83-85 by a rejoinder by Ruch, Knight, and Lutes, still maintaining meagre sampling and inadequate computation as defects in the study.

This article also contains summaries of the work of Winch, Mead and Sears, Taylor, Beatty, Ballard and McClelland.

Comment: Johnson finds both addition-carrying and addition-borrowing called Austrian. He quotes an unnamed author whose classification is as given at the beginning of this chapter and then uses numbers I, II, III, and IV respectively.

Johnson, J. T.

Efficiency of Decomposition Compared With That of Equal Additions as a Technique in Subtraction of Whole Numbers. Mathematics Teacher 24:5-13. January 1931.

Assuming that there is no issue as to additive or subtractive methods of subtraction, since students use both in the same example (subtractive for easy and additive for harder), the problem was whether to increase the subtrahend or decrease the minuend.

Four groups of University graduates and former teachers numbering 89, six groups of Normal College students totaling 140, and twelve groups of pupils in grades five, six, seven, and eight totaling 404,participated in this experiment. In all, 1386 tests were given to 693 subjects of whom 355 used the equal additions method and 338 the borrowing method. The test contained the 100 primary facts and thirty examples.

Four tables show the results in accuracy and in time required. Subtracting the per cent of accuracy in the examples from the per cent of accuracy in the facts resulted in a loss in accuracy due to technique, of 13.13% in borrowing and 9.78% in carrying. The difference therefor in favor of carrying was 3.35%. The difference in time was fifty-six seconds.

The author concludes that the carrying method is 3.3% more accurate and 14.3% faster but recognises the need for experiments on a larger scale.

Kramer, Sister Mary Immaculata

Permanence of Improvement and the Distribution of Learning in Addition and Subtraction.

Doctor's Dissertation, Catholic University of America, Washington, D.C. 1931.

See Chapter Fifteen on Drill.

3d + 4th Grs.
rept. Supt.

LIBRARY SUMMARIES

Ruch, G.M. and Mead, Cyrus D.
A Review of Experiments on Subtraction.
Twenty-ninth Yearbook of the National Society for the Study of Education : 671-678. 1930.

The principal methods are summarized to include (1) the complementary method, (2) the borrow and repay method, (3) the take-away method, and (4) the additive method.

The experimental studies made in England, viz. those of Ballard, McClelland, and Winch are summarized as well as experiments made here by Mead and Sears, Taylor, Johnson, Buckingham, Beatty, and Osburn.

Eight results are given in terms of the studies under consideration:

1. The results in the Winch, McClelland, Mead and Sears, and Johnson studies are uncertain because of the small number of cases.

2. The English studies and that by Osburn find the equal-additions superior to the decomposition. England has not raised the question of the additive versus the subtractive methods.

3. In the Mead and Sears investigation, which is the first evidence on additive versus subtractive, equality is the only conclusion.

4. An interesting result in the Mead and Sears study is the

negative transfer and the tendency to add.

5. In the Taylor investigation 95% of the pupils abandoned the additive method by sixth grade. Beatty confirms this finding.

6. The results found by Taylor and by Beatty tend to support the contention of Ruch, Knight, and Lutes that the method used by Johnson was vitiated by selection, that the poorer students might change and the additive method be superior.

7. Buckingham's material is worthy of consideration. Of seven comparisons six favor the take-away over the additive.

8. Osburn differentiates between two types of take-away methods and finds the decomposition superior to the additive and the equal-additions inferior to the additive.

Two final conclusions, in general, are that to settle the question as to superiority of methods investigation should follow the pupils year by year through the elementary school, high school, and in higher mathematics, and that the differences must be small or they would have been discovered before. The authors also suggest that the investigation should include subtraction of fractions, mixed numbers, and compound numbers since total superiority and not superiority in one phase needs to be known.

Board of Education of the City of New York
Educational Measurements for the Class Teacher. #11 Inventory of Errors and Remedial Suggestions - Addition and Subtraction.

See Chapter Five for Addition.

Irmina, Sister M.
The Relative Merits of the Methods of Subtraction.

Catholic University of America, Educational Research Bulletin
Vol. 3 #9. 1928. Catholic Education Press. Washington, D.C.

Four subtraction meanings or ideas are listed. Descriptions are given of the subtractive, additive, and complementary methods analyzed by Osburn.

Studies by Mead and Sears, Buckingham, Knight, Ruch and Lutes, Stone, Ballard, McClelland, Winch, Taylor, Beatty, Johnson, and Osburn are summarized.

A bibliography of 32 articles is listed.

DISCUSSION

Gibson, Charles S.
Some Methods in Subtraction.
Mathematics Teacher 17:486-494. December 1924.

This article discusses three types of subtraction problems or meanings. Five methods are discussed with arguments for each. The methods are: addition carrying, addition borrowing, decomposition, equal additions, and the complementary method.

Ruch, G. M. and Knight, F. B. and Lutes, O. S.
On the Relative Merits of Subtraction Methods - Another View.
Journal of Educational Research 11:154-155. February 1925.

In this news item the authors question the findings of Johnson, offering these criticisms: (1) Of the five distributions only one was based upon enough samplings to warrant conclusions; (2) The set of examples was too easy having 35% of the scores perfect; (3) The differences are lessened by

Catholic University of America, Educational Research Bulletin Vol. X No. 1935. Catholic Education Press. Washington, D.C.

Four subtraction theories or types are listed. Descriptions are given of the subtrahend, additive, and complementary methods employed by Beatty.

Studies by Mead and Sears, Buckingham, Wright, Ruch and Lutes, Winch, Ballard, McClelland, Winch, Taylor, Beatty, Johnson, and Osburn are summarized.

A bibliography of 36 articles is listed.

PERIODICALS

Gibson, Theodore H.
Some Methods in Subtraction.
Mathematics Teacher 17:457-463. December 1924.

This article discusses three types of subtraction problems at beginning. Five methods are discussed with arguments for each. The methods are: addition carrying, addition borrowing, decomposition, equal additions, and the complementary method.

Ruch, G. M. and Knight, F. B. and Lutes, O. S.
On the Relative Merits of Subtraction Methods - Another View.
Journal of Educational Research 11:102-109. February 1925.

In this test after the methods equated the efforts of teachers, following these conclusions: (1) Of the five distributions only one was based upon enough sampling to warrant conclusions; (2) the set of examples was too easy after 87 in the second semester; (3) the differences are limited by correcting the calculation error; (4) Since 80% learned the decomposition method the four small groups had undergone selective tendencies. They also state that the only conclusion that can be drawn is that statistically no genuine differences were found.

Knight, F. B. Ruch, G. M. and Lutes, O. S.
How Shall Subtraction be Taught?
Journal of Educational Research 11:157-168. March 1925.

Thirteen criteria are offered for the selection of a subtraction method. A refers to subtractive methods, B to additive, C refers to addition-borrowing, D to Austrian or addition-carrying. The problems are superiority of A or B and superiority of C or D.

1. Which offers the most helpful transfer? A and C are equal to if not superior to B and D. B saves Subtraction facts but loses the subtraction idea.

2. Which avoids negative transfer and interference? Finds A better because of these difficulties with the additive: (a) There is confusion with change making; (b) In $\begin{array}{r}9\\ \underline{7}\end{array}$ thinking 9 last may lead to writing 9; (c) The association with addition may in $\begin{array}{r}9\\ \underline{7}\end{array}$ lead to the answer 16 instead of 2; (d) In examples like $\begin{array}{r}2\frac{1}{4}\\ \underline{1\frac{1}{4}}\end{array}$ where the difference in the fractions is zero there may be difficulty since zero never occurs in the additive method; (e) In the additive method 1 is added which may lead to the habit of always carrying 1.

3. Which uses the simplest process? Type B is simplest with the first advantage lessening and disappearing as more complicated numbers appear.

correcting the calculation errors; (4) that the improved the concentration during the four weeks groups not reduced the relative inequality. They also state that the only conclusion that one can draw is that statistically no genuine differences were found.

Knight, F. B., Ruch, G. M. and Lutes, O. S.
How shall subtraction be taught?
Journal of Educational Research 11:157-168. March 1925.

Thirteen criteria are offered for the selection of a subtraction method. A refers to subtractive borrowing, B to subtractive, C refers to addition-borrowing, D to additive-carrying. The problems are superiority of A or B and superiority of C or D.

1. Which method has more helpful transfer? A and B are equal to, if not superior to C and D. D saves subtraction facts but loses the subtraction idea.

2. Which avoids negative transfer and interference? Here A better because of fewer difficulties with the additions. (a) there is confusion with change making; (b) in 2 changing 6 just may lead to writing 9; (c) the association "all additions" in 2 lead to the common 16 instead of 11; (d) in examples like 12 where the difference in the quantities in the subtrahend may be difficulty where zero (0) occurs in the additive method; (e) in the additive method D the extra work may lead to the habit of always carrying 1.

3. Which uses the simplest process? Type C is simplest since the first subtraction learning and subtraction is more conditioned certain types.

4. Which carries contributions to the other aspects of the total subject? C does better than D because (a) It makes use of number relations, e.g. $\begin{array}{r}52\\ \underline{17}\end{array}$ the 52 becomes 40 and 12 the form changing without a change in value; (b) In subtraction of mixed numbers C involves 4¼ = 3 5/4 which is closely allied to reducing fractions in answers and changing mixed numbers to improper fractions in multiplication and division.

5. Which calls for mastery of the fewest facts? Types A and C at least even with B and D.

6. Which can be carried furthest with reasonable ease? The Austrian gets more difficult the farther it goes. It is justified if Austrian division is used but it isn't.

7. Which can be adequately rationalized? The children can see A.

8. Which provides the most effective drill? It isn't known whether effective drill for hard spots in each type is possible or not.

9. Which contributes best to thinking in problem solving? Type A in the subtractive or remainder type of problem, the additive in the additive type of problem.

10. Which has a better check? Addition and Austrian no check. Thinks adding should be reversed as a check.

11. Which is best illustrated by object teaching? Same as seven.

12. Which is most natural? Any habit seems most natural.

13. Which is defended by experimental evidence? The best research is by Mead and Sears which favors A and C.

Summary. Prefers A and C, but recognises that any decision is based on theory.

Johnson, J. T.
Still on the Relative Merits of Subtraction Methods.
Journal of Educational Research 12:80-83. June 1925.

This is an answer to the Ruch, Knight, Lutes article of February 1925. In it the author points out misinterpretations made by those criticizing him. He also argues for time as a variable.

Ruch, G. M., Knight, F. B., and Lutes, O. S.
A Rejoinder to Professor Johnson's Criticisms.
Journal of Educational Research 12:83-85. June 1925.

The authors still maintain that only one of the distributions reported in Johnson's investigation was large enough for a sampling and that the calculation was inadequate.

Stone, J. C.
How We Subtract.
Benj. Sanborn, Boston. 1926.

In the first chapter of this monograph the author discusses three methods of subtraction, viz. equal additions, decomposition, and additive, their meanings and uses.

Early methods of subtracting by use of the abacus and of the scratch method are discussed. The types of subtraction present in early English and American texts are mentioned.

The studies by Ballard, McClelland, Winch, Mead and

Sears, Greene, and Johnson are summarized.

The last chapter is devoted to suggestions for teaching the additive method, advantages of which were set forth in previous chapters.

Colwell, Lewis W.
Teaching Subtraction by Building.
American Childhood 13:24-26. 1927.

This article lists twelve problems of the additive type to show that subtraction does not always occur in situations demanding lessening.

Arguments, presented for the additive method, include the loss of time in learning the primary subtraction facts.

Upper decade drill in subtraction is given with the facts not limited to the 75 needed in short division although division is mentioned.

Myers, G. W.
The Minus Sign in Vertical Subtraction in Arithmetic.
School Science and Mathematics 28:627-633. June 1928.

This article contains arguments for not discontinuing the use of the minus sign in vertical subtraction: (1) Almost all teachers and texts have long been using the sign in this way; (2) It is customary to employ the other symbols +, x, and ÷; (3) The complete recasting of old arithmetical points of view is the intrinsic duty of algebra after subtraction has been absorbed into the enlarged concept of addition in algebra; (4) No one objects to its use in 364-182.

Sylvester, Harold D.

A Diagnostic Test in Subtraction.

Normal Instructor and Primary Plans 38 #4:54 & 109. February 1929.

A twenty step subtraction test is given which can be used with any method.

Myers, G. C.

Hard Subtraction.

Primary Education - Popular Educator 46:544. March 1929.

General suggestions for remedial work in subtraction are given in this article.

SUMMARY OF CHAPTER SIX

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Taylor, J.S. 1919	Subtraction method used by children after the addition carrying had been official for six years.	11,368
	Buckingham B.R. 1927	Comparative efficiency of additive versus take-away method of teaching the subtraction facts.	110 pairs
	Osburn, W.J. 1927	Analysis of methods, summary of important studies and report of methods used by children in grades four, five, and six.	1414
	St. Louis Public Schools 1928	Methods of subtraction prescribed by large cities. Methods used by children in grades eight and nine in St. Louis.	1065
Minor Research	Mead, C.D. and Sears, I. 1916	Use of additive subtraction and multiplicative division.	2 second grades
	Beatty, W.W. 1920	Subtraction methods used by children who were taught the addition carrying method.	83

	Johnson, J.T. 1924	Subtraction methods used by students in Chicago Normal College.	477
	Johnson, J.T. 1931	Study to determine whether to increase the subtrahend or decrease the minuend in subtraction.	693
Library Summaries	Ruch, G.M. and Mead, C.D. 1930	Summary of studies made by Ballard, McClelland, Winch, Mead and Sears, Taylor, Johnson, Buckingham, Beatty, and Osburn.	
	Board of Education of City of New York 1929	Summary of sections pertaining to addition and subtraction from "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.	
	Irmina Sister M. 1928	Description of four subtraction methods analyzed by Osburn. Summaries of studies by Mead and Sears, Buckingham, Knight Ruch and Lutes, Stone, Ballard, McClelland, Winch, Taylor, Beatty, Johnson, and Osburn.	
Discussion	Gibson, C.S. 1924	Discussion of three types of subtraction problems or situations. Five subtraction methods and arguments for each.	
	Ruch, G.M. Knight, F.B. and Lutes, O.S. 1925	Criticisms of Johnson's findings in his 1924 article.	
	Knight, F.B. Ruch, G.M. and Lutes, O.S. 1925	Thirteen criteria for selection of a subtraction method.	
	Johnson, J.T. 1925	An answer to the Ruch, Knight, Lutes article of February 1925.	
	Ruch, G.M. Knight, F.B. and Lutes, O.S. 1925	An answer to the Johnson article in the same issue.	
	Stone, J.C. 1926	Discussion of present and early subtraction methods. Summaries of studies by Ballard, McClelland, Winch, Mead and Sears, Greene, and Johnson. Suggestions for teaching the additive method.	

[illegible], [illegible]. 1930	Subtraction methods used by students in [illegible] colleges.	177
[illegible], [illegible]. 1931	Study to determine whether to increase the achievement in [illegible] grades the methods [illegible] subtraction.	225
[illegible], [illegible] and [illegible], 1930	Summary of studies made by Ballard, [illegible], [illegible], and [illegible], Taylor, Johnson, [illegible], [illegible], and others.	
[illegible] Com- mittee of [illegible] of New York 1939	Summary of studies pertaining to addition and subtraction facts [illegible] in New York Public Schools [illegible] [illegible] in the [illegible] grades.	
[illegible] [illegible] 1935	Description of four subtraction methods analyzed by others. Summaries of studies by [illegible], Buck- ingham, Knight, [illegible], [illegible], [illegible], [illegible], [illegible], Taylor, [illegible], [illegible], and others.	
[illegible], [illegible]. 1934	Discussion of three types of sub- traction problems [illegible], [illegible] methods and [illegible].	
[illegible], [illegible], and [illegible], 1935	Criticism of [illegible]'s findings in the [illegible] article.	
[illegible], [illegible], and [illegible], [illegible]. 1936	Thirteen criteria for selection of a subtraction method.	
[illegible], [illegible] 1936	An answer to the [illegible], [illegible] article of February 1936.	
[illegible], [illegible], [illegible] 1936,	An answer to the [illegible] article in the [illegible] issue.	
[illegible], 1937	[illegible] of [illegible] and [illegible] methods. [illegible] of studies by [illegible], [illegible], [illegible] and [illegible], and [illegible], [illegible] for [illegible] the [illegible] method.	

Colwell, L.W. 1927	Discussion of the additive type of problem, arguments for the additive method, and suggestions for upper decade drill in subtraction.
Myers, G.W. 1928	Arguments for not discontinuing the use of the minus sign in vertical subtraction.
Sylvester, H.D. 1929	A twenty step subtraction test.
Myers, G.C. 1929	General suggestions for remedial work in subtraction.

CONCLUSIONS

The problem of the selection of a subtraction method received most consideration, forming the subject matter of ten studies. Taylor and Beatty found what method was used by children in New York and San Francisco respectively after several years in which the addition-carrying method was official . The addition method had been abandoned for borrowing methods by over 50% of the children tested.

The studies by Osburn and Johnson and that made in St. Louis attempt to discover the efficiency as well as the nature of the methods in use. The studies by Johnson agree with Osburn's study as to the efficiency of equal additions or carrying methods in comparison with the borrowing methods but the author feels that more work is necessary to determine which carrying method is preferable. The study in St. Louis finds the take-away-carrying most efficient.

Further work on this phase of subtraction includes discussions by Knight, Ruch, and Lutes based on thirteen criteria for the selection of a subtraction method and by

Stone on the advantage of the additive method, summaries by Ruch and Mead and by Sister Irmina of the experiments relating to subtraction methods.

Closely related to the above problem is the desirable method for teaching the primary facts. The studies both by Mead and Sears and by Buckingham show the take-away form to be in favor but neither study is conclusive. In spite of the continued attack on this problem for more than a decade there is still more opinion and theoretical analysis than convincing scientific evidence. If in the first place a uniform number of methods were chosen for experimentation, four probably (although the complementary method may not be receiving as much attention as it should), and if uniform names were agreed upon, experiments then carried on should study not only immediate results with small groups but results of controlled teaching with large groups, these groups being tested at intervals at least through the grades and junior high school. Possibly since experimentation over years has not shown conclusive superiority of one method over another there is no great difference. In which case determination and adoption of the most used method might take place.

Other articles on subtraction are general, adding little or nothing to the information of the teacher.

Chapter Seven

MULTIPLICATION

MAJOR RESEARCH

Clemens, Paul B. and Neubauer, Paul F.
A Supervision Project in Multiplication.
Journal of Educational Research 18:387-396. December 1928.

The purpose of this investigation was to study the errors and to provide remedial measures in multiplication.

The subjects were 2000 children in grades four B to eight A.

An analysis of the types of examples, which could be made using one-place multipliers and two and three-place multiplicands with all carrying and zero difficulties, resulted in 42 types of situations. Test cards were constructed which contained examples of these 42 types. Three cards or one hundred twenty-six examples comprised a test. Four tests were given at intervals of increasing length, the results of which were tabulated.

The authors'findings are as follows:

1. Specific drill during the special help period for two weeks reduced the per cent of error from 7.6 to 0.8.
2. There were several zero difficulties. Zero in the units place gave greater difficulty than zero in the tens place.
3. Certain errors, such as zero in the units place, persisted in spite of the specific drill given.
4. The greatest error in carrying was made when the sum of the

carried figure and the product was in the decade above the product, for example $\begin{array}{r}881\\ 7\\ \hline 7167\end{array}$

5. Examples involving carrying were not of equal difficulty. Carrying 1 twice caused 2.88% of the errors while carrying 8 twice caused 16.58%.

6. Drill upon specific weakness proved better than indiscriminate drill.

7. Indiscriminate drill was wasteful because the pupils used skills in which they were proficient instead of needed skills.

8. The diagnostic cards showed the weakness and the drill cards provided the drill.

9. Forgetting played an important part.

10. The causes of failure in multiplication were: (1) Errors in upper decade addition facts, (2) Carelessness, (3) Errors in multiplication combinations, (4) Omission of the carrying figure, (5) Confusion of numbers, (6) Transposition of figures in the product, (7) Adding instead of multiplying, (8) A disregard of zero when in the tens place.

Norem, G. M. and Knight, F. B.
The Learning of the One Hundred Multiplication Combinations. Report of Society's Committee on Arithmetic. Twenty-ninth Yearbook of the National Society for the Study of Education. 1930.

See Chapter Four on Relative Difficulty of Number Facts.

MINOR RESEARCH

Thorndike, Edward L.
The Effect of Continuous Exercise and of Rest Upon Difficult Mental Multiplication.
Journal of Educational Psychology 5:597-599. December 1914.

72 undergraduates multiplied mentally two three-place numbers, working from 100 to 140 minutes with a rest of 30 minutes and with a night's rest. The thirty minute rest period increased the product per unit of time by 5%, the night's rest increased the product 7% more, the accuracy being the same.

This is primarily a fatigue study with no great value for arithmetic.

Painter, W. S.
Efficiency in Mental Multiplication Under Extreme Fatigue.
Journal of Educational Psychology 7:25-30. January 1916.

The author worked from 11.00 P.M. to 3.07 A.M. recording his reactions from hour to hour. He found an abrupt end to his ability to work, not a gradual loss of ability with continued ability to do less difficult computations.

This study has value as study of mental fatigue but not from viewpoint of arithmetic, since mental multiplication of a four-place number by a four-place number is useless.

Fowlkes, John Guy
A Report of a Controlled Study of the Learning of Multiplication by Third Grade Children.
Journal of Educational Research 15:181-189. March 1927.

The problem in this study was (1) to teach the 100 multiplication facts by the use of a text with the teacher doing as little talking as possible; (2) to use printed directions and devices for remedial work instead of oral instruction; and (3) to ascertain the relative difficulty of the primary facts developed by pupils individually when the frequency of drill on the facts is the same.

Thirty-one children in the first half of the third grade continued the experiment over twenty days using the Fowlkes-Goff Book I PartI. Of the 35 minutes daily devoted to this work, 20 minutes were used for developing ten new facts by the mimeographed text; 10-15 minutes were used for ten repetitions of each fact developed; and 5-10 minutes were used for problem work. On the eleventh day the 100 facts were given, the remaining time being devoted to problems. From the twelfth day through the twentieth day the children worked for ten minutes with the facts which they had failed on the previous day, using the remaining time for problems.

A table of relative difficulty is given but there are no suggestions as to the factors considered.

The author concludes that there are grave doubts as to the difficulty of zero facts. He finds text-book material organized in sufficient detail is more effective than the usual oral presentation accompanied by inadequate text material. Less difference than expected was found in the difficulty of the facts. The author sees the need for more pages in texts, for drill material based on the method of teaching instead of the frequency of errors, and for controlled studies of the basic facts.

SUMMARY

Board of Education of the City of New York.

Educational Measurements for the Class Teacher #12, January 1930.

Inventory of Errors and Remedial Suggestions - Multiplication and Division.

This bulletin is a summary of the sections pertaining to multiplication and division of "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.

DISCUSSION

Jones, Hilton Ira and Jones, Blanche P.

A Little Understood Principle in Multiplication.

School Science and Mathematics 25:36-43. January 1925.

This article describes Phillip's system of multiplication based on the graphic method for $(a + b)^2$.

Lewis, George

The Multiplication Bugbear.

American Childhood Vol. 13 #3:16-18 & 58. November 1927.

This article describes devices used by a teacher of slow children. These activities include jumping and relay games and the store. The subject matter involved is multiplication "tables".

Colwell, Lewis

Shortening the Multiplication Table.

American Childhood Vol. 13 #4;31-33 & 78. December 1927.

This article suggests the elimination of 69 facts

through limiting the multiplication tables to 9x9 instead of 12x12. A list of the 100 primary facts is given. The author advocates the inclusion of the zero facts to be learned independently as the ones also may be learned. He advocates teaching the reverses of each fact in a table as each table is taught thereby lessening each remaining table.

Shelton, M. R.

Teaching Multiplication Combinations.

American Childhood 16:44. May 1931.

Individual work is suggested, the teacher writing the numbers from 1-12 horizontally in mixed order and the children copying, placing a 2 below and multiplying.

SUMMARY OF CHAPTER SEVEN

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Clemens, P.B. and Neubauer 1928	Study of errors and remedial measures in multiplication.	2000
Minor Research	Thorndike, E.L. 1914	Effect of continuous exercise and of rest upon mental multiplication. Psychological value.	72
	Painter, W.S. 1916	Effect of fatigue upon mental multiplication, Psychological value.	1
	Fowlkes, J.G. 1927	Use of author's text to replace teacher direction and explanation.	31
Library Summaries	Board of Education of City of New York 1930	Summary of sections pertaining to multiplication and division in "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.	

Discussion	Jones, H.I. and Jones, B.P. 1925	Description of Phillip's system of multiplication based on the graphic method for $(a + b)^2$.
	Lewis, G. 1927	Suggestions for devices for multiplication drill.
	Colwell, L. 1927	Suggestions for teaching 100 primary facts instead of the 144 facts in the tables.
	Shelton, M.R. 1931	Suggestions for multiplication drill.

CONCLUSIONS

Little work of value to the teacher of arithmetic Methods has been done in the field of multiplication. The study by Clemens and Neubauer of errors and their correction in the use of one-place multiplication is the best contribution relating to this topic. The summary by the Board of Education of New York City of Miss Lazar's investigation is also helpful.

Multiplication, like addition, has tempted psychological experimentation as seen in the work of Thorndike and Painter. Neither study involves socially useful material.

The study by Fowlkes has no general interest dealing as it does with the use of a small number of subjects.

The discussions by Lewis and by Shelton relate to devices for teaching multiplication. Jones and Jones discuss a little used multiplication method and Colwell suggests teaching 100 primary facts instead of the tables.

No one has published any research comparing the results, in terms of accuracy and time saving, of teaching facts in tables with the teaching of unrelated groups. There are still many school systems in which tables are being taught making it

difficult to convince students in a teachers college that facts can be taught in any but table order. Although not doubting the results I would like to see a controlled experiment of this nature.

Chapter Eight

DIVISION

MINOR RESEARCH

Rogers, Don C.
Supervising a Class in Long Division.
Chicago Schools Journal 7:296-300. April 1925.

The problem was to determine means for bringing an eighth grade up to standard in the fundamentals, especially division.

The Courtis Series B test was given and the Courtis Practice Pads were used.

The author finds that the principal errors in division are (1) misplacing the decimal point in the quotient, (2) omitting the zero in the quotient, and (3) underestimating the trial divisor.

Remedial work included: (1) changing the quotient from side to top, (2) writing the quotient figure directly over the dividend figure. (3) writing a quotient figure every time a number was brought down, (4) practicing placing the decimal point without doing the computation, (5) practicing the primary uneven division facts, (6) using the first two figures for a trial divisor, (7) watching for a repeated quotient figure, and (8) drawing a line from the dividend figure to the new place.

The results of this remedial work show a class increase in accuracy of two grades and in speed of two and three-fourths grades.

Comment: Data on test results are incomplete.

Chapter Eight

DIVISION

MINOR RESEARCH

[illegible], Edna G.

Supervising a Class in Long Division.

Chicago Schools Journal V:298-300. April 1923.

The problem was to determine means for bringing an eighth grade up to standard in the fundamentals, especially division.

The Courtis Series B test was given and the Courtis Practice Pads were used.

The author finds that the principal errors in division are (1) misplacing the decimal point in the quotient, (2) omitting the zero in the quotient, and (3) underestimating the trial divisor.

Remedial work included: (1) changing the quotient from side to top, (2) writing the quotient figure directly over the dividend figure, (3) writing a quotient figure every time a number was brought down, (4) permitting placing the decimal point without using the explanation, (5) practicing the primary inverse division facts, (6) using the first two figures for a trial divisor, (7) watching for a correct quotient figure, and (8) drawing a line over the dividend figure in the new place.

The results of this remedial work show a great improvement in accuracy of the pupils and in speed of work and better-quality grades.

[illegible]

John, Lenore
The Effect of Using the Long-Division Form in Teaching
Division by One-Digit Numbers.
Elementary School Journal 30:675-692. May 1930.

The problem was to compare the results from teaching the long-division form for one-digit numbers with the results from teaching the short-division form.

Two fifth grades in the University of Chicago School participated, having the same teacher, the same amount of time, and the same material. One-digit divisors were taught before two-digit divisors.

A description is given of the construction of the six tests, two of which contained one-digit divisors and four of which contained two-digit divisors.

Twelve tables give the results of the group tests and of the individual tests.

Group I, taught by the short-division form, worked with greater speed and Group II, taught by the long-division form, worked with greater accuracy before two-digit divisors were taught. After divisors of two or more digits were taught the long-division pupils were faster and more accurate in the solution of any examples. Those who were taught both forms used the long-division form for all examples.

It is further found in the individual tests that pupils who were taught both forms of division made more errors and a greater variety of errors than did the pupils who were taught only the long-division form.

The author feels that the number of children does not

justify more than tentative conclusions. Although the results were in favor of teaching the long-division form there may be times when the short-division form is more economical, e.g. in reducing fractions to lowest terms or improper fractions to mixed numbers. Therefore she suggests that the long-division form be taught first for all examples, that the short-division form be taught as a short cut to be applied to one-digit divisors, to two-digit divisors ending in zero, to three-digit divisors ending in two zeros, and to divisors which are familiar multiples such as 12 and 25.

Grossnickle, Foster E.
How to Estimate the Quotient Figure in Long Division.
Elementary School Journal 32:299-306. December 1931.

The apparent-quotient method employs the first figure of a two-digit divisor as the trial divisor, the increase-by-one method increasing the first figure by one when the second figure is 5 or more.

In this study examples with two-figure divisors and two-figure quotients were used. An apparent-quotient of 10 or more was considered as nine.

Three tables show the results of the analysis. From tables 2 and 3, page 304, the following comparison has been made:

	Apparent-Quotient	Increase-by-one
Apparent quotient is true quotient	63.94%	78.23%
True quotient is one less than apparent quotient	32.28%	12.27%
True quotient is two less than apparent quotient	2.93%	.54%
True quotient is three less than apparent quotient	.64%	.10%

From his analysis the author finds that the true quotient may be five away from the apparent-quotient but can never be more than three away in using the increase-by-one rule. The apparent-quotient rule requires but one method of procedure while the increase-by-one requires two. In the apparent-quotient rule the change from apparent to true quotient is always a decrease, in the increase-by-one rule it may be an increase or a decrease. In the apparent-quotient rule the product of the divisor and the estimated quotient is larger than the partial dividend, in the increase-by-one rule it may be smaller.

LIBRARY SUMMARY

Board of Education of the City of New York

Educational Measurements for the Class Teacher #12. January 1930. Inventory of Errors and Remedial Suggestions - Multiplication and Division.

See Chapter Seven on Multiplication.

DISCUSSION

McFarland, Blanche B.
Arithmetic in the Intermediate Grades, - A Study of Division.
Primary Education 34:122-138. February 1926, and 34:192-193.
March 1926.

This article discusses the difficult facts, twenty-one points involved in division, and six phases of remedial work.

The second article continues the discussion of division giving the steps in the transition from short to long division, calling attention to division by measuring and by partition.

Johnson, J. T.
Short Division or Long Division First.
Chicago Schools Journal 12:55-56. October 1929.

Two values in teaching long-division before short-division are (1) the elimination of the need for memorizing the 360 uneven facts, the upper decade subtraction facts, and the remembering of the remainder, and (2) the foundation provided for long-division with two-place divisors. Short-division is a short process and should be treated as such, the author says.

Colwell, Lewis W.
Better Grading of Long Division Material.
Chicago Schools Journal 12:193-195. January 1930.

Long-division examples are divided into two cases, those with apparent quotients and those with non-apparent quotients. Case I is subdivided into examples with no

carrying, with carrying in the multiplication, borrowing in the subtraction, carrying and borrowing, with remainders, with three-figure quotients, with an intermediate zero in the quotient and with a final zero in the quotient. Case II is subdivided into examples with the apparent quotient figure diminished by one, by two, and by three; with a final zero and with a teen divisor.

Myers, G. C.
Teaching Long Division.
Grade Teacher 47: 542 & 576-577. March 1930; 47:626 & 654. April 1930; 794 & 831. June 1930; 47:44 & 73. September 1930; and 47:290 & 325. December 1930.

These magazines contain a series of discussions on the teaching of long division, including review of facts, dramatization of meaning of division, use of models for long-division, steps, teaching long before short-division, teaching new steps in long division still using one-place divisor, grading long-division examples with a two-place divisor, use of the apparent-quotient rule with suggestions for estimate quotient figure when apparent quotient is not true quotient, and emphasizing the essential points in long-division.

Grossnickle, Foster E.
How to Test the Accuracy of the Estimated Quotient Figure.
Elementary School Journal 32:442-446. February 1932.

The guide figure is the tens digit in a two-figure divisor. This tens digit may be given place value and then

used as a guide number as in $25\,\overline{)1374}$ when the question asked is the number of 20's in 137. Or the numerical value of the tens digit may be used as a guide figure. In this latter case the question asks the number of 2's in 13.

Children, when they meet this type, should be able to test without writing instead of writing and erasing the product as is the usual custom.

The procedure is suggested for the use of the tens digit when given place value. In the example $24\,\overline{)1387}$ think "There are how many 20's in 138? Answer 6 and a remainder of 18. Since six 4's are 24, a number greater than 18, the quotient must be one less than 6, or 5". The author further states on page 443 that in order to estimate 20's, 30's, 40's, etc., and to state the remainder in a given partial dividend the pupil must be able to respond automatically to

1. $20\,\overline{)60}$ - divisor a factor of the dividend.
2. $20\,\overline{)62}$ - remainder is the units digit of the dividend.
3. $20\,\overline{)70}$ - remainder a multiple of 10.
4. $20\,\overline{)72}$ - remainder a combination of types 2 and 3.

This suggestion is given for using the numerical value of the tens digit as a guide figure. In this example $34\,\overline{)1685}$ think "There are five 3's in 16 and one remaining which makes the next number 18. Will 18 contain six 4's? Since six 4's are more than 18, the true quotient will be one less than 6, or 5." (The true quotient will be 4 and not 5.)

The author also suggests that the estimated quotient and remainder be written on the margin at the right of the example.

$$\begin{array}{r l} 37 & \\ 38\overline{)1406} & 4,\ 20R \\ \underline{114} & \\ 266 & 8,\ 26R \\ \underline{266} & \end{array}$$

We are told that if the estimated figure is two or more removed from the true quotient figure, the procedure for checking the accuracy of the estimated figure is not easily applied.

Excluding 13-18 as divisors there are only 1211 examples having two-figure divisors and two-figure quotients, or 2.72% of the possible number of examples, in which the true quotient is more than one removed from the estimated figure. These examples can be done by actually finding the product of quotient and divisor.

The author recommends the "apparent method", the second described above, since there is little difference with respect to the percentage of corrections necessary to find the true quotient and since the method for checking the validity of an estimated quotient figure is easier.

Grossnickle, Foster E.

Classification of the Estimations in Two Methods of Finding the Quotient in Long Division.

Elementary School Journal 32:595-604. April 1932.

In an earlier article the author has shown that 44,550 estimations are possible with a two-figure divisor and a quotient of not more than two figures. This includes all divisors from 11 through 99 except 20, 30, 40, etc. When the apparent method is used for all divisors in which the units digit is from 1 through 8 and when the nine two-figure numbers ending in 9 are increased to the next decade number there are 31,061 examples in which the estimated quotient is the

true quotient and 13,489 examples in which the estimated quotient must be corrected. In some of these latter examples the need for correction is more easily determined than it is in others.

The following classification, given on pages 595 and 596 is based upon the ease of estimation and determination of need for correction:

Classification of Estimations When Apparent Method is Used.

1. Quotient self-evident, as in 24)$\overline{26}$
2. Estimated quotient is true quotient - 24)$\overline{50}$
3. True quotient is one less than estimated figure but the need for correction is evident - 24)$\overline{65}$
4. True quotient is one less than estimated figure but the need for correction is not evident - 24)$\overline{92}$
5. True quotient is two or more removed from the estimated quotient - 24)$\overline{185}$
6. Divisors are 13-18 inclusive. (These are later called "division demons").

Table I, containing a "Classification of the Estimations of the Quotient by the Apparent Method for Two-Figure Divisors, the Division Demons and Divisors Ending in Nine Being Omitted", is summarized in Table II, on page 600, as follows:

tive quotient and I.Q. are [illegible], to [illegible] the relation-ship need be corrected. In none of these latter examples the need for correction is more [illegible] significant than it is in others.

The following classification, used on pages [illegible] and [illegible] is based upon the sense of [illegible] and the determination of need for correction:

Classification of relationships when reported ratings is used.

1. Correction self-evident, as in [illegible]
2. Extremely deficient in true quotient - [illegible]
3. True quotient is, on [illegible], more than estimated figure but the need for correction is evident - [illegible]
4. True quotient is [illegible] less than estimated figure but the need for correction is not evident - [illegible]
5. True quotient is [illegible] of [illegible] figure [illegible] correction evident - [illegible]
6. [illegible] and [illegible] evident. These are those called "[illegible]"

Table 7, [illegible] a [illegible] of the relations of the quotient in [illegible], [illegible] the [illegible], the [illegible] and [illegible] in the [illegible]. As [illegible] in Table 7, [illegible] as follows:

Classification	Number of possible examples	Per cent
1. Quotient self evident	4,200	10.96
2. Estimated quotient true quotient	22,052	57.56
3. True quotient 1 less than estimated but need for correction evident	4,366	11.40
4. True quotient 1 less than estimated but need for correction not evident	6,983	18.23
5. True quotient 2 or more removed from estimated	709	1.85
6. Total requiring estimating, sum of 2, 3, 4, and 5	34,110	89.04
7. Total requiring estimating, correction not evident, sum of 4 and 5	7,692	20.08
8. Sum of 1 and 6	38,310	100.00

The significance of 7692 examples, in which the true quotient is not the apparent quotient and in which the need for correction is not evident, is great enough in the author's opinion to merit consideration by the teacher and by the text-book writer.

The following "Classification of Estimations When the Increase-by-one Method is Used", given on page 600, involves examples in which the tens digit is increased by 1 when the units digit is six or more, and in which 10 is used as a trial divisor for divisors ending in 9;

1. Quotient self-evident - 26)$\overline{281}$
2. Estimated quotient is true quotient - 26)$\overline{62}$
3. True quotient 1 more than the estimated figure - 26)$\overline{82}$
4. True quotient 2 more than the estimated figure - 26)$\overline{235}$
5. Divisor is a division demon - 16)$\overline{142}$

Table III which contains a "classification of Estimations for Two-Figure Divisors Ending in 6.7 and 8 - Increase-by-one Method - Division Demons Omitted" is summarized in Table IV on page 603:

Classification	Number of possible examples	Per cent
1. Quotient self-evident	4,200	10.96
2. Estimated quotient true quotient	25,656	66.98
3. True quotient 1 more than estimated quotient	3,142	8.20
4. True quotient 2 more than	8	0.02
5. True quotient 1 less than estimated but need for correction evident	2,781	7.26
6. True quotient 1 less than estimated but need for correction not evident	2,415	6.30
7. True quotient 2 less than estimated quotient	108	0.28
8. Total requiring correction which is not evident - sum of 3, 4, 6 and 7	5,673	14.81
9. Total number of examples	38,310	100.00

The author concludes that the apparent method gives results almost as accurate as the increase-by-one if divisors ending in 9 are treated as a special case. In the increase-by-one two distinct procedures are required to make the necessary co rrections because the true quotient may be 2 more or 2 less than the estimated quotient. In the apparent method the correction is always a decrease requiring but one procedure.

Early 1925

Summary 1929

SUMMARY OF CHAPTER EIGHT

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Rogers, D.C. 1925	Study of errors and remedial work in division with an eighth grade.	An eighth grade
	John, L. 1930	Results from teaching long-division form for one digit divisors compared with results from teaching short-division form.	Two fifth grades
	Grossnickle, F.E. 1931	Analysis of examples with two-figure divisors and two-figure quotients using both apparent-quotient and increase-by-one rules.	
Summary	Board of Education of the City of New York 1930	Summary of sections pertaining to multiplication and division in "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.	
Discussion	McFarland, B.B. 1926	Difficult facts, points involved in division, and phases of remedial work. Steps in transition from short to long-division. Division by measuring and by partition.	
	Johnson, J.T. 1929	Suggests that teaching long-division form before short-division form would eliminate learning of 360 primary uneven division facts and would provide a foundation for long-division form with two-place divisors.	
	Colwell, L.W. 1930	Analysis of long-division examples to provide better grading of material.	
	Myers, G.C. 1930	General discussion of various phases of the teaching of long-division.	
	Grossnickle, F.E. 1932	Suggestions for testing the accuracy of the estimated quotient figure.	
	Grossnickle, F.E. 1932	Classifications of 38,310 examples showing ease of determination of quotient figure by apparent-quotient and increase-by-one rules.	

CONCLUSIONS

The first problem in division to receive consideration, that of the value in teaching the long-division before the short-division form was suggested by Johnson in a discussion published in 1929. The following year Miss John's experiment, resulting in favor of teaching the long-division form, took place. As the number of children involved in this experiment was small a repetition of it with a larger number of subjects would justify more than tentative conclusions.

The next problem studied was the use of the apparent-quotient and increase-by-one methods for determining the quotient figure in division with two-place divisors. Grossnickle, in three articles compares the two methods, gives directions for testing the accuracy of the estimated quotient figure and classifies the estimations on the basis of the ease with which they are determined. His conclusions favor the apparent-quotient method since the results are almost as accurate as the increase-by-one method if the divisors ending in 9 are treated as a special case and since the apparent-quotient correction is always a decrease requiring but one procedure. While this analysis is very helpful in presenting the whole detail of division with two-place divisors the final decision as to the better method must come from classroom experiment. Colwell in an earlier article suggested better grading of long-division material but not with the detailed analysis accorded the topic by Grossnickle. Suggestions for remedial work on the basis of discovered errors are given by Rogers and by the

Board of Education of the City of New York in a summary of the work of Miss Lazar. Discussions by Myers and McFarland are general in nature.

Chapter Nine

COMMON FRACTIONS

MAJOR RESEARCH

Kallom, Arthur W.

Determining the Achievement of Pupils in Common Fractions. Boston Public Schools. School Document #5, 1918. Bulletin #15. Department of Education - Investigation and Measurements.

This study involved 3513 children in grades six, seven, and eight in ninety-five grade classes in Boston. Seven tests in multiplication and division of common fractions were constructed containing such examples as 1/8 x 12, $\begin{array}{r}246\ 1/5\\ \underline{5}\end{array}$,

4 7/8 x 1/8, $\begin{array}{l}32\ 1/3\\ \underline{69\ 1/2}\end{array}$, $\frac{3}{4} \div 8$, 5678 1/3 ÷ 5, 3/5 ÷ 1/3.

Analysis of errors is first reported for grades seven and eight because of the low degree of accuracy which is due to lack of knowledge in grade six. Each test is analyzed and types of errors noted. The findings are set forth in nine tables.

The author finds that the range of variation in the medians of the ten school districts tested extends from 0 - 92%. A large per cent of the pupils show an utter lack of knowledge of the processes in multiplication of mixed numbers and division of mixed numbers by integers and integers by mixed numbers. This the author infers is due to lack of drill. In the tests in division the chief source of error is in the apparent inability of the individual pupil to distinguish between the

dividend and the divisor. The large variation within the class is due to the ineffectiveness of class instruction, the author infers, and holds important the remedial work based on individual needs.

Comment: The author states that an effort has been made to keep the tests within the realm of the practical and that the terms of the fractions were kept small. In the light of research made prior to 1917 no doubt these tests seemed practical in nature. But in view of Wilson's investigation of socially useful processes, published in 1918, and of his later studies, the usefulness of 7/9 in the multiplication of fraction and integer may be questioned as well as the occurrence in life situations of multiplication of mixed numbers and of division involving fractions and mixed numbers.

Brueckner, Leo J.
Analysis of Errors in Fractions.
Elementary School Journal 28:760-770. June 1928.

This article contains an analysis of 21,065 errors made by children in grades five A, six B, and six A in the solution of examples in the fraction processes.

The examples used in this study were constructed after an analysis which resulted in the discovery of forty types of examples in addition, forty-five in subtraction, forty-five in multiplication, and thirty-seven in division. The first five types of examples are given for each process. In addition these first five types are $\begin{array}{r}1/3\\ \underline{1/3}\end{array}$ $\begin{array}{r}1/8\\ \underline{5/8}\end{array}$ $\begin{array}{r}1/4\\ \underline{3/4}\end{array}$ $\begin{array}{r}4/5\\ \underline{2/5}\end{array}$ $\begin{array}{r}3/4\\ \underline{3/4}\end{array}$

The differences in their difficulty are based on the treatment

of the answer. The first five examples in multiplication are 1/4 x 8, 1/9 of 3, 2/5 x 2, 2/3 x 16, 5/6 x 14.

Five tables show the number of errors analyzed for each process; the analysis of errors in addition of fractions, in subtraction of fractions, in multiplication of fractions, and in division of fractions.

The author's findings are as follows:

1. Errors in computation made up 13.8% of the addition errors, 8.2% of the subtraction errors, 28.7% of the multiplication errors, and 13.8% of the division errors.
2. The major causes of errors in each process are listed.
3. The major difficulties in all processes were lack of comprehension of the process involved, failure to reduce fractions to lowest terms, and failure to change improper fractions to whole or mixed numbers.
4. Changing fractions to common denominators gave little difficulty in addition but gave much trouble in subtraction.
5. Changing mixed numbers to improper fractions in multiplication and division caused difficulty.
6. The kinds of errors analyzed existed in all grades.
7. This study was based on written work and should be supplemented by individual study of the work of children with special difficulties.
8. Attention should be given to the preparation of special exercises to eliminate the major difficulties.

Comment: The social value of the test as a whole can be estimated only from the sample examples given and from examples used to illustrate specific errors. Both the integers and the terms of the fractions are small numbers. But the question

arises as to the possible social usefulness of 187 types of examples in common fractions.

Neal, Elmer A. and Foster, Inez
An Experiment with Remedial Work in Common Fractions.
Elementary School Journal 29:280-283. December 1928.

The problem in this investigation was the maintenance of skill in the fundamentals with common fractions through drill materials provided for the teachers.

An analysis of types of examples in common fractions was made but the number of types discovered is not stated. Seven samples of addition types are given of which five are the same as Brueckner's first five examples in addition. The three samples given for multiplication show fraction multiplied by fraction with no cancellation, with one cancellation, and with two cancellations.

After an analysis of fractions, diagnostic tests with a 5 minute time limit were formulated. Three practice sheets followed each test and one review followed every two diagnostic tests.

Test results noted by the author on page 283 are as follows:

	February		May		
Stanford Achievement	Number of Pupils	Median	Number of Pupils	Median	Gain
Control	397	108	385	124	16
Experimental	639	112	640	136	24
Informal Test					
Control	417	76	413	84	8
Experimental	647	80	629	92	12

The authors conclude that organized practice material in the hands of the children is more effective in teacher time, and pupil time, and in final results than teacher provided material.

Comment: Doubtless no one will question the need for organized practice materials, in common fractions, to be placed in the hands of the children. However, one may question the use of the Stanford Achievement Test which so far exceeds the limits of social usage.

Polkinghorne, Ada Ruth
The Concepts of Fractions of Children in the Primary School.
Master's Thesis 1929. University of Chicago.

The purposes of this study were to determine (1) whether children have certain concepts of fractions; (2) if so, what these concepts are; (3) when the children acquire these concepts; and (4) how they acquire them.

Tests were given to 266 children of the University Elementary School. These children ranged in chronological age from 4½ to 11 years and in mental age from 5 to 12 years,

The tests were made up as follows:

I. Unit fractions

A. Part of a single object, ½, ½, ¼, ½, 1/3.

B. Part of a group of objects, ½ of 2, 1/3 of 3, ½ of 4, 1/3 of 6, ¼ of 4.

C. Comparison of two objects, 1 ft. equals 1/3 yd., ¼ of a line, ½ of 4 2, ¼ of 8 2.

D. Comparison of two groups of objects, 2 ½ of 4, 3 1/3 of 9.

II. Proper fractions that are not unit fractions.

A. 2/3 of 1, ¾ of 1.

B. 2/3 of 3, ¾ of 4, 2/3 of 6, ¾ of 8.

C. 1 stick 2/3 of another, one line ¾ of another.

D. 2 peaches in basket 2/5 of all so 3 3/5 of 5 and 6 2/3 of 9.

III. Improper fractions.

A. 3/2 of 1, 4/3 of 1.

B. 3 3/2 of 2, 4 4/3 of 3.

C. This stick 3/2 of that and so x 4/3 of 4.

D. 5 5/4 of 4, 7 7/5 of 5.

IV. Identification of fractions.

A. How many ½'s in 1? 1/3's in 1? ¼'s in 1? 1/5's in 1? 1/8's in 1?

B. Read 2/2, 3/3, 4/4, 5/5.

C. 3/2 ? 4/2 ? 6/3 ? 8/2 ? 8/4 ? 9/3 ?

D. ? 4/2, 6/2, 6/3, 8/2, 8/4, 9/3.

E. ? 3/2, 4 /2, 4/3, 5/4, 7/5.

V. Equivalent fractions.

A. ½ how many 4ths, 6ths, 8ths, 12ths.

B. 2/3 how many 6ths, 9ths, 12ths.

C. 9/12 how many 4ths?

D. 8/12 how many 3rds?

These tests were given by the same examiner in the testing laboratory, on a schedule that contained no time element, care being taken to express satisfaction with the child's work without telling him whether his answer had been right or wrong. Both responses in the form of objective expression and of numerical expression required an explanation of how the children arrived at their conclusions unless their actions made their method of thinking perfectly obvious. Children's answers were recorded verbatim.

Chapter Four contains descriptive statements concerning the results, Test items are repeated with illustrations of satisfactory and unsatisfactory answers.

Data are presented in twelve tables and six figures. Table I (pages 57-80) shows the responses of each child to each item in the test.

The author draws the following conclusions:

1. The primary children who took these tests have certain concepts of fractions. (The median number of satisfactory responses was 15 for the first series and 21.5 for the second series. 1/3 of all responses were satisfactory.)

2. Children as a group have acquired four important concepts of fractions: unit fractions, proper fractions other than unit fractions, improper fractions, and identification of fractions. (1/3 of all responses satisfactory.)

3. Children as a group have not acquired the concept of equivalent fractions. (Only one child able to make a satisfactory response.)

4. Children know more about unit fractions than about any other fractions included in the test. ($\frac{1}{2}$ of responses satisfactory.)

5. Children know less about proper fractions other than unit fractions than they do about unit fractions. (1/5 to 1/3 of responses satisfactory.)

6. Children know very little about improper fractions and about identification of fractions. (Less than 10% of responses satisfactory.)

7. Children know more about a unit fraction that is used as a part of a single object than about any other of the fractions used in the test. (37% of the satisfactory responses in the first series and 16% of the satisfactory responses in the second series were made to this exercise.)

8. Next to a unit fraction which is used as a part of a single object, the children know a unit fraction that is used in the

comparison of two objects. (In the second series 16% of all satisfactory responses were made to this exercise.)

9. Children know less about a unit fraction that is used as a part of a group of objects, than about a unit fraction that is used in the comparison of two objects.

10. Children know less about a unit fraction that is used in the comparison of two groups of objects than they do about a fraction of this type that is used in the comparison of two objects.

11. One-half is the best known fraction and it is known best when it is used to designate a part of a single object.

12. Children know $\frac{1}{4}$ better than 2/4 or $\frac{3}{4}$ but they do not know it as well as $\frac{1}{2}$.

13. Children know very little about 1/3 and 2/3, 2/5 and 3/5.

14. It is easier for the children to express their concepts objectively than to express them numerically.

15. The first concept of fractions acquired by these children is the concept of unit fractions which began to be acquired in the 4-6 year group.

16. The concepts of proper fractions other than unit fractions and even the concept of improper fractions began to be acquired by the children of the 6-8 year group.

17. Some of the children began to identify fractions between their 8th and 10th years.

18. The greatest gain in the acquisition of the various concepts of fractions included in these tests come for these children at the third grade level, or between their 7th and 10th years.

Comment: A very helpful study of concepts of fractions acquired by children through their home, school and outside school experiences and not as a result of teaching.

James, Blanche
A Special Study of Diagnosis in Fractions.
Master of Arts Thesis. University of Minnesota. July 1930.

The author quotes Brueckner, Probst, Uhl, Monroe, Osburn, Souba, and Gist on the need for diagnosis of fractions. She also surveys the literature in this field and summarizes the work of Buswell and Judd, Brueckner, Souba, Smith, Uhl, Woody, Monroe, and Philips.

The problem in this study was to (1) analyze the mistakes and difficulties, (2) make a classified catalogue of mental errors, and (3) list the frequency of errors.

The Brueckner Tests for Addition, Subtraction, Multiplication, and Division of Fractions were given four times from September 1928 to June 1930 to the children of grades five A, six B, and six A. Over 100 children, chosen because of their outstanding weaknesses, were studied.

The technique used in the study included discussion of the case with the teachers and principal, uninterrupted work in a quiet room, establishment of friendly relations with the children to be tested, work on only one process at a time to avoid fatigue, explanation that the purpose was not testing but remedial, and careful recording of results usually in the child's own words.

The 68 boys and 37 girls taking part in this study

ranged in age from 10 to 17 years, with most of their ages 11, 12, or 13.

An analysis of the errors in addition shows the following:

I. Lack of comprehension of the process involved 205.
  a. adding numerators 13.
  b. adding denominators 9.
  c. adding numerators and adding denominators 10.
  d. adding whole number and fraction 26.
  e. adding whole number and mixed number 10.
  f. adding whole number, fraction, and mixed number 21.
  g. adding mixed number and fraction 19.
  h. adding mixed numbers 33.
  i. adding fractions 43.
  j. inverting fraction part of answer 5.
  k. multiplying numerators 9.
  l. multiplying numerators, multiplying denominators, and adding the products 3.

II. Reducing fractions to lowest terms 64.
  a. fractions not reduced 35.
  b. denominator divided by numerator 16.
  c. denominator and numerator divided by different numbers 5.
  d. fraction reduced but whole number discarded 8.

III. Difficulties with improper fractions.
  a. improper fractions not changed to mixed numbers 6.
  b. improper fractions changed to mixed numbers but the mixed numbers not added 20.
  c. numerator divided by denominator with other errors 17.
  d. failure to write integral part of answer 3.

IV. Computation errors 40 (addition 13, subtraction 12, multiplication 5, and division 10).

V. Omitted 16.

VI. Wrong operation 25.

- a. fractions added, whole numbers multiplied 2.
- b. fractions added, whole numbers subtracted 2
- c. fractions subtracted, whole numbers added 2.
- d. subtracted 2.
- e. multiplied 2.
- f. added denominator, disregarded numerator 7.
- g. added common denominator to numerator 5.

VII. Partial operation 11.

- a. added only the fractions in the mixed numbers 5.
- b. added two fractions, discarded third 5.
- c. added only the whole numbers in mixed numbers 9.

VIII. Changing fractions to common denominators 92.

- a. changed to wrong denominator 46.
- b. didn't know how to find common denominator 13.
- c. disregarded a denominator of more than one 5.
- d. divided common denominator by denominator and added result to numerator of fraction 5.
- e. Multiplied the numerator by the product of the denominator and the common denominator 2.
- f. inverted fractions with like numerators 8.
- g. added ¼ + ½ + ¼ and called answer 1/10 3.
- h. 2/3 + ¾ = 18/12 + 9/12 = 27/12 = 2¼ 5.
- i. 1/8 + 5/8 = 8/8 + 5/8 = 13/8 = 1 5/8 5.

IX. Difficulty in borrowing 44.

- a. when adding fraction and whole borrowed from whole 16.

b. adding mixed and whole 13.

c. adding whole and fractions 7.

d. adding whole and mixed 5.

e. 3 3/5 + 2 4/5 = 2 13/5 + 2 4/5 = 2 17/5 = 5 2/5   3.

X. Errors in copying 11.

XI. Difficulty with proper fractions 6.

In subtraction of fractions the main types of difficulties were:

I. Difficulties in borrowing 202.

II. Wrong process 58.

III. Difficulties in reduction

IV. Lack of comprehension of process involved 119.

V. Computation errors 33.

VI. Difficulty in changing to similar denominators 89.

VII. Omitted 58.

VIII. Partial operation 17.

IX. Errors in copying 8.

X. Zero difficulties 78.

In multiplication the principal types of errors were:

I. Computation errors 74.

II. Lack of comprehension of process involved 255.

III. Difficulty in reduction 50.

IV. Omitted 30.

V. Changing improper fractions to mixed numbers 56.

VI. Changing mixed to improper fractions 61.

VII. Errors in copying

VIII. Difficulties in cancellation 94.

IX. Difficulties with signs 8.

X. Changing the order of multiplier and multiplicand 1.

In division the principal types of errors were:

I. Wrong operation 161.

II. Computation errors 84.

III. Lack of comprehension of process involved 88.

IV. Difficulty in reducing fractions to lowest terms 58.

V. Changing mixed numbers to improper fractions 79.

VI. Omitted 11.

VII. Changing improper fractions to mixed numbers 53.

VIII. Zero difficulties 4.

IX. Difficulties in cancellation 74.

X. Difficulties with signs 27.

XI. Wrong or unnecessary steps 3.

XII. Errors in copying 8.

The author's conclusions are as follows: (1) General drill and unintelligent working of problems will not eradicate difficulties, (2) Written group tests show the frequency of specific difficulties but not the reason for their existence, (3) Many pupils have difficulties the nature and cause of which appear only as the pupil is individually tested, (4) Efficient teaching gets at the difficulties and applies remedies, (5) The interest and cooperation of the children must be sought in eradicating the errors, (6) Low I.Q.'s should be taken into account, (7) The home and school histories should be given consideration.

A bibliography of 23 references is supplied. The appendix contains the Brueckner Tests.

Comment: This is a very thorough study but adds little not already given by Brueckner.

MINOR RESEARCH

Kallom, Arthur W.

Analysis of and Testing Common Fractions.

Journal of Educational Research 1:177-192. March 1920.

The fundamental processes in fractions are classified, on pages 178 to 180, as follows; an explanation of each step being given:

A. Similar fractions

1. 2/5 + 1/5 = 3/5.
2. 4/7 + 4/7 = 8/7 = 1 1/7.
3. 5/9 + 1/9 = 6/9 = 2/3.
4. 5/6 + 5/6 = 10/6 = 5/3 = 1 2/3.

B. Dissimilar fractions

a. L.C.D. is largest denominator

5. ½ + 3/8 = 7/8.
6. 2/5 + 7/10 = 11/10 = 1 1/10.
7. ½ + 3/10 = 8/10 = 4/5.
8. ¾ + 5/12 = 14/12 = 7/6 = 1 1/6.

b. Common denominator equals the product of the denominators

9. 2/5 + 3/8 = 13/40.
10. 7/9 + ¼ = 37/36 = 1 1/36.

c. Common denominator found by factoring

11. 1/6 + 2/9 = 7/18.
12. ¼ + 5/6 = 13/12 = 1 1/12.
13. 1/6 + 2/15 = 9/30 = 3/10.
14. 1/6 + 9/10 = 32/30 = 16/15 = 1 1/15.

The above analysis holds for addition but for subtraction examples 2, 4, 6, 8, 10, 12, and 14 would be omitted.

The steps in multiplication included in this analysis are: fraction times whole number, mixed number times whole number, whole number times fraction, whole number times mixed number, fraction times fraction, mixed number times fraction, and mixed number times mixed number.

The steps in division are: fraction divided by whole number, mixed number divided by whole number, whole number divided by fraction, whole number divided by mixed number, fraction divided by fraction, and mixed number divided by fraction.

The results of testing in the fifth grades in five districts in Boston lead to these conclusions: (1) There was an increase in accuracy and speed as the children worked the successive forms, (2) Testing like this showed the ability of the children to add fractions, (3) Children may have no trouble with examples not requiring reduction and have difficulty with reduction, (4) The time required varied from 2 to 41 minutes on twelve examples.

Suggestions for diagnosis are (1) Make a thorough study of errors, (2) Find how the pupil did his work, (3) Plan corrective work on the basis of (1) and (2), and (4) Keep a record of each pupil's performance.

Comment: More space is devoted to the analysis of types of examples than to the presentation of data to support conclusions.

Morton, R. L.

An Analysis of Pupil's Errors in Fractions.

Journal of Educational Research 9:117-125. February 1924.

36 eighth grade pupils were given, in November, 2174 examples of which 663 were incorrectly solved. 35 pupils were given, in April, 2305 examples, of which 366 were incorrectly solved.

Four tables give the errors in addition, subtraction, multiplication, and division. A discussion of these errors follows the tables.

The conclusions of the author are: in 91.3% of the cases the causes of the errors were detected, the three principal causes being inadequate conception of processes involved, confusion of the operations, and lack of an adequate degree of skill in the fundamental operations with integers.

Comment: No information appears concerning nature of examples.

Kee, Olive A.

An Analysis of Errors in Operations with Common Fractions.

American Education 30:302-305. May 1927.

An analysis was made of the results of the seventh grade and some sixth grade children in one town from the Massachusetts State Wide Tests of 1925 and 1926.

Tables are given for 1925 and 1926.

The author's results show that errors in addition were: adding given numerators and adding given denominators; adding given numerators but keeping the larger denominator; adding the given numerators but keeping the smaller denominator; adding given numerators and multiplying given denom-

inators; adding new correct numerators but omitting denominators.

Errors in general were: failure to reduce, straight substitution of one process for another; confusion within the process itself, a general confusion of rules - the processes blurred, and poor reduction, cancellation or raising to higher terms.

Comment: This study based on Wilson's General Survey Test reports only those errors occurring in socially useful fraction material.

LIBRARY SUMMARY

Board of Education of the City of New York
Educational Measurements for the Class Teacher. #13.
March 1930. Inventory of Errors and Remedial Suggestions - Common Fractions.

This bulletin is a summary of the sections pertaining to Common Fractions of "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.

DISCUSSION

Wildeman, Edward
The Teaching of Fractions.
Plymouth Press, Chicago. pp. 145. 1925.

This discussion contains a general analysis of the teaching procedure in common fractions for the most part keeping content within social usage. No direct results of scientific studies are discussed nor is there detailed analysis in terms of unit skills.

Myers, George W.

The Norm of Social Utility in Common Fractions.

School Science and Mathematics 27: 734-740. October 1927.

The theme of this article is an argument for the inclusion of fractions not socially useful.

Operations of high social utility of which the sixth grade should have reasonable control are said to be: division of any whole number by any three or four-figure divisor reducing the remainder to lowest terms, division by any simple mixed number checking the procedure, calculation of the average of a resonable number of magnitudes checking the work, reduction of ordinary fractions to decimal or percentage equivalents.

Knight, F. B.

Analyzing the Distribution of Drill in Fractions.

Journal of Educational Method 8:205-211. January 1929.

Brueckner, in the Journal of Educational Method 7: 352-358 May 1928, discussed Knight's method of analyzing fractions arguing for the analysis of "types of situations" in which fractions occur instead of specific or "unit skills" of which each example is composed.

This article is a reply to the Brueckner article. Knight points out that Brueckner has assumed an analysis of subtraction from the analysis of division and calls to attention that this assumption was not necessary because an analysis of subtraction had been made. He states that both the skills which are found and the situations in which they are

found are important. Brueckner has described total situations in terms of elements. The fundamental issue, he thinks, is the choice of one of these alternatives:

1. Teach all total situations which we want the learner to cope with - impossible except with a narrow function.
2. Stress teaching the elements which once learned will operate in any complex situation - impossible for children to see similarities as we see them.
3. Teach all the elements of a total mental function each one in enough varying gross total situations so that the child will receive adequate practice in the different combinations of elements.

Osburn, W. J.
Estimating the Difficulty of Exercises in Common Fractions by Job Analysis.
Educational Research Bulletin (Ohio State University) 8: 12-14. January 9, 1929.

Job analysis to determine the relative difficulty of fraction processes to replace the present method of right and wrong responses is suggested in this article.

A part of the table which gives samples of the difficulty of 10 examples in common fractions is repeated here.

I.	1/3 + 1/3	1 + 1	A
II.	4/5 - 2/5	4 - 2	S
III.	½ of 4	4 ÷ 2	D
IV.	6 ÷ ¼	invert, 4x6	I. M.
V.	½ + ¼	4 ÷ 2, 1x2, 2 + 1	D. M. A.

Brueckner, Leo J. and Kelly, Fred
A Critical Evaluation of Methods of Analyzing Practice in Fractions.
Report of Society's Committee on Arithmetic. Twenty-ninth Yearbook of the National Society for the Study of Education. 525-534. 1930.

This discussion is based on Knight's "unit skills" in subtraction. Table 1 contains the "unit skills" in subtraction, Table 2 lists sixteen examples containing all of Knight's unit skills, Table 3 reports the number of times each unit skill occurs in the examples in Table 2, and Table 4 lists Brueckners 53 "types" with skills occuring and the per cent of error.

The conclusions are that an analysis of the frequency of unit skills is no indication of the extent of practice on the various combinations of these skills.

Comment: In May 1928 Brueckner criticized Knight's "unit skills" in much the same way inferring subtraction treatment from Knight's division skills. In January 1929 Knight replied calling to attention that he had analyzed subtraction. He also said that Brueckner's "types" or "situations" are based on unit skills which must be taught and practiced in many situations. This article adds nothing new except that the criticism is now based on Knight's subtraction analysis of 1925 to which he called Brueckner's attention in the previously mentioned article.

Laughlin, Butler

Teaching Arithmetic in Grades V and VI.

Chicago Schools Journal 12:369-373. May 1930.

The author advocates that we teach things people are in the habit of using, things people would use if they knew how, and things necessary to make people quantitative thinkers.

In common fractions he suggests (1) the fractional concept, (2) addition and subtraction of fractions, although he states that there is little need for these, (3) multiplication and division to include especially multiplication with one factor a fraction since there is not so much use for a fraction multiplied by a fraction or a mixed number multiplied by a mixed number. He also feels that it may not be any advantage to deal with fractions apart from integers.

Miller, G. A.

Inverting the Denominator of a Fraction.

School Science and Mathematics 30: 881-883. November 1930.

This article discusses the early appearance of the inverting of the divisor in the mathematical literature of India. The author states that this feature did not occur in the mathematical literature of Europe before the 16th Century.

Miller, G. A.

Sexagesimal Fractions.

School and Society 33:58-59. January 10, 1931.

The article discusses sexagesimal fractions used by early Babylonians. The author sees special sexagesimal fractions as instances of survival of fittest in an effort to overcome the difficulties inherent in the use of common fractions.

Miller, G. A.
On the History of Common Fractions.
School Science and Mathematics 31:138-145. February 1931.

As the title of the article implies, the development of common fraction symbols in early civilizations is discussed.

Knight, F. B.
A Note on the Organization of Drill Work.
Journal of Educational Psychology 16:108-117. February 1925.

Relates to the evaluation of text-books.

Brueckner, Leo J.
A Technique for Analyzing the Distribution of Drill in Fractions.
Journal of Educational Method 7:352-358. May 1928.

Relates to the evaluation of text-books.

Contains criticism of Knight's "unit skills".

Beall, Ross H.
What is Good Fraction Drill?
Elementary English Review 6:49-52. February 1929.

This article deals with an analysis of three series of texts for the addition of fractions.

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Kallom, A.W. 1918	Analysis of errors in seven tests in multiplication and division of common fractions.	3513
	Brueckner, L.J. 1928	Detailed analysis of 21,065 errors in the fraction processes.	
	Neal, E.A. and Foster, I. 1928	Use of organized practice materials in fractions.	2100
	Polkinghorne A.R. 1929	Results of s test to discover fraction concepts in children from 4½ to 11 years.	266
	James, Blanche 1930	Analysis of difficulties based on individual oral diagnosis.	100
Minor Research	Kallom, A.W. 1920	Analysis of fundamental processes in fractions and results with these in Boston schools.	Fifth grades in five Boston districts
	Kee, O.A. 1927	Analysis of errors in fractions made on Massachusetts State Wide Tests of 1925 and 1926.	Sixth and seventh grade children in one town
Library Summary	Board of Education of the City of New York 1930	Summary of the sections pertaining to common fractions of "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.	
Discussion	Wildeman, E. 1925	General analysis of the teaching procedure in common fractions.	
	Myers, G.W. 1927	Argument for including fractions not socially useful	

Knight, F.B. 1929	A reply to an article by Brueckner in the debate over "unit skills" and "situations".
Osburn, W.J. 1929	An analysis of examples in common fractions in terms of the processes involved and the frequency of their occurrence.
Brueckner, L.J. and Kelly, F. 1929	Criticism of Knight's analysis of subtraction of fractions in terms of "unit skills".
Laughlin, B. 1930	Suggestion of simplification of common fraction computation on basis of social need.
Miller, G.A. 1930	Discusses the early appearance of the inverting of the divisor in mathematical literature.
Miller, G.A. 1931	Discussion of sexagesimal fractions and their early use.
Miller, G.A. 1931	Development of common fractions in early civilizations.
Knight, F.B. 1925	Evaluation of text-book material on fractions.
Brueckner, L.J. 1928	Evaluation of text-book subject matter and criticism of Knight's "unit skills".
Beall, R.H. 1929	Analysis of three series of texts for the addition of fractions.

CONCLUSIONS

Tests to discover types of errors occurring in fractions make up the largest group of studies. Of these studies the most detailed is that by Miss James involving individual testing of over 100 children showing weaknesses in work in fractions. A long list of errors with the frequency of occurrence of each includes types of errors discoverable more easily through individual than class testing.

Kallom in two studies presents results of tests given in the Boston schools covering multiplication and division of fractions and all processes in fractions. These early studies suggest the need for general diagnostic and corrective work and are forerunners of detailed analyses which were later conducted by others. Morton in 1924, Kee in 1927, and Brueckner in 1928 analyzed pupil errors agreeing in general in their findings with the results obtained by James and by Kallom. A summary of errors and remedial work is contained in the bulletin published by the Board of Education of New York City based on the work of Miss Lazar.

Analysis of the elements present in computation with fractions is discussed by Osburn who lists for each example the amount of addition, subtraction, multiplication, and division required in its solution. Brueckner in 1928 criticizes Knight's use of "unit skills" in analyzing fraction computation urging "types" or "situations" as a better basis for analysis. Knight in January 1929 replies to Brueckner's article and Brueckner and Kelly in the Twenty-ninth Yearbook reply in turn to Knight continuing the discussion. With all due respect to these specialists in this field, one who is searching for help in determining what subject matter to teach and how to teach this subject matter to children could wish for less argument over theoretical analyses. If the theory of social utility were applied to fractions a few classroom experiments would probably suffice to show how this simplified subject matter might best be taught.

Perhaps the study of the concepts of fractions gained by children as a result of experience not of teaching is a curriculum rather than methods study. But as in the consideration of informal number it is difficult to make a sharp distinction between the information learned by children and the situations in which it is learned.

An experiment by Neal and Foster show the advantages of organized practice material in the hands of the children over teacher planned material.

General discussions by Wildeman, Laughlin; an argument by G. W. Myers for the inclusion of fractions not socially useful; three articles by Miller on the history of fractions; and articles on the evaluation of text-books by Knight, Brueckner, and Beall complete the contributions in the field of fractions.

The outstanding need in common fractions is a drill service based on social utility in which the analysis will be simple and primarily dictated by use.

Chapter Ten

DECIMAL FRACTIONS

MINOR RESEARCH

Drushel, J. Andrew

A Study of the Amount of Arithmetic at the Command of High School Graduates Who Have Had No Arithmetic in their High School Course.

Elementary School Journal 17:657-661. May 1917.

This article reports results of a test given to 459 students in public or private schools in or around St. Louis. These two examples constituted the test: 400 ÷ .66 2/3 + 876.16 ÷ .296 and 66.6264 ÷ 4.

Two methods for placing the decimal point in the quotient in division of decimal fractions were in use by the students. Method A consisted in subtracting the number of places in the divisor from the number of places in the dividend, Method B consisted in making the divisor a whole number.

The average accuracy of the papers on which method A was used was 66. The average accuracy by method B was 99. An examination of all arithmetic papers for these students for twenty weeks showed 100% accuracy for those using method A but method B failed 25% of the time.

Monroe, Walter S.

The Ability to Place the Decimal Point in Division.

Elementary School Journal 18:287-293. December 1917.

This experiment had for its purpose analysis of the ability to place the decimal point in division of decimal fractions.

A test composed of eighteen examples of different types in which the answers were computed requiring only the placing of the decimal point, was given to three groups of pupils in sixth, seventh, and eighth grades. Four tests were given including as divisors integers, tenths, hundredths, and a variety of the three.

Five tables present the results.

The author concludes that the decimal point is not placed in quotients by means of a general rule or general ability but by means of several specific abilities.

Brueckner, Leo J.

Analysis of Difficulties in Decimals.

Elementary School Journal 29:32-41. September 1928.

The problem was to determine the types of examples with which pupils have difficulty and the reason for errors on each type.

A test of 121 items on the meaning, addition, subtraction, multiplication, and division of decimal fractions was given to 300 pupils in grades six, seven, and eight in the Minnesota schools.

Comment: Some multiplication and division items exceed social usage.

The results show 114 different kinds of errors with the largest variety of errors in grade eight. The kinds of errors were more numerous in multiplication and division than in addition and subtraction. Errors in reading, writing, and conversion of decimal fractions numbered 33, in addition 15, in subtraction 14, in multiplication 26, and in division 26. There was no evidence that errors were typical by grades.

Five tables list types of errors noted above.

The author's conclusions are:

1. Inadequate concepts of numerical values was equalled only by the misplaced point in multiplication and division.
2. Frequent misspelling of the decimal in word form occurred, e.g., hundreds for hundredths.
3. Placing the decimal point caused twice as many errors in addition than inaccuracies in adding.
4. In subtraction borrowing and placing the decimal point were the causes of errors.
5. Misplacing and omitting the decimal point were errors in multiplication. There were many inaccuracies in multiplying.
6. In division the errors were misplacing and omitting the decimal point, zero placement and inaccuracies in dividing.

LIBRARY SUMMARY

Board of Education of the City of New York

Educational Measurements for the Class Teacher. #14 April 1930.

Inventory of Errors and Remedial Suggestions - Decimals.

Summary of sections pertaining to Decimals from "Diagnostic and Remedial Work in Fundamentals in the Intermediate Grades" by May Lazar.

DISCUSSION

Wildeman, Edward

The Teaching of Fractions, Chapter V.

Plymouth Press, Chicago. 1925. pp. 145.

This chapter discusses Arabic notation, decimal fraction notation, use of decimal versus common fraction, reading and writing of decimal fractions, addition and subtraction of decimal fractions, multiplication of decimal times integer, decimal times decimal, and decimal times 10, 100, 1000, etc.; division of decimal by integer, decimal by 10, 100, 1000, etc., decimal by decimal; expressing quotient approximately and placing the decimal point in the quotient.

Arleigh, M. W.

Arithmetic in Cartoons.

Educational Screen 8:22-24. January 1929.

The value of the cartoon, especially in the upper grades, is suggested in this article. In decimal fractions the decimal point is crowned king and bestows titles according to place and position. A series of cartoons from day to day as the topic is developed affords visual aid in an interesting manner. It is suggested that everything unimportant be eliminated and that crude drawings be used. Four illustrations accompany the discussion.

Wright, Robert

The Division of Decimals - A Method in Outline.

The School 19:752-754. April 1931.

The author suggests expressing the example as a common fraction, e.g., $\frac{.00013041}{.567}$, changing the fraction to contain one place integer in denominator, $\frac{.0013041}{5.67}$ and then dividing 5.67 ÷ .0013041, placing point by inspection.

SUMMARY OF CHAPTER TEN

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Drushel, J.A. 1917	Comparison of accuracy resulting from placing the point in division of decimals by subtracting number of places in divisor from number of places in dividend with that resulting from making the divisor a whole number.	459
	Monroe, W. S. 1917	Attempt to determine whether the decimal point is placed in quotients by means of a general ability or by means of several specific abilities.	3 grades
	Brueckner, L.J. 1928	Analysis of types of errors occurring in a test covering all phases of decimal fractions.	300
Library Summary	Board of Education of the City of New York 1930	Summary of the sections pertaining to decimal fractions of "Diagnostic and Remedial Work in Arithmetic Fundamentals for Intermediate Grades" by May Lazar.	
Discussion	Wildeman, E. 1925	General discussion of decimal fractions.	
	Arleigh, M.W. 1929	Suggestions for the use of cartoons in teaching the meaning of decimal fractions.	
	Wright, R. 1931	Suggestion for expressing the divisor with one integer and placing the point by inspection.	

CONCLUSIONS

No major researches have been made in the field of decimal fractions.

Probably the method of placing the point by making the divisor a whole number, found by Drushel to be more accurate than subtracting number of places in divisor from number of places in dividend, has since his study in 1917 become quite generally accepted. Wright's suggestion for placing the point in division by inspection deserves consideration but his examples used for illustration go beyond social usage.

Studies of errors in decimal fraction computation have been determined by Brueckner. The summary published by the Board of Education of New York City also deals with this phase of decimal fractions.

A discussion by Arleigh of the use of cartoons is suggestive of further use of visual aids. There is some question however if drill is kept behind meaning and if meaning is developed through use whether the more or less artificial aids are necessary.

The general discussion by Wildeman completes the studies in this field.

No investigations have been published relative to the value of a certain amount of drill in placing decimal points in multiplication and division when the computation in the drill examples is done leaving only the placing of the point to be done. Nor are there published results of

investigations pertaining to the placing of the decimal point in multiplication and division by inspection instead of by rule. Both of these features might well receive consideration.

Chapter Eleven

PERCENTAGE

MINOR RESEARCH

Kircher, H. W.

Study of Percentage in Grade VIII A.

Elementary School Journal 27:281-289. December 1926.

A test containing ten items was given to 133 children in Grade Eight A in a school in Sheboygan.

The test items, listed on page 281, were as follows:

1. One cent is what part of one dollar?
2. Find one-half per cent of two dollars.
3. Change one-half per cent to a common fraction.
4. Change 7/5 to per cent.
5. Five cents is five per cent of what?
6. One cent is what per cent of two dollars?
7. 75% of a number is 6. What is the number?
8. 110% of a number is 220. What is the number?
9. 5½% of a number is 44. What is the number?
10. A man lost 15% of his money and saved the remainder. What per cent of his money did he save?

Data are set forth in four tables.

The author concludes that there is no clear conception on the part of 80% of the Grade Eight A children included innthis study with regard to the ppinciples of percentage but a confusion of whole numbers, decimal and common fractions with percentage. He infers that the time of

the pupils is wasted in working problems in a formal way instead of applying the principles of percentage and further says that percentage as now taught is largely a guessing exercise.

Edwards. Arthur
A Study of Errors in Percentage.
Report of the Society's Committee on Arithmetic. Twenty-ninth Yearbook of the National Society for the Study of Education. 621-640. 1930.

The author, on page 621, says of this error study that it contains neither defense for the social utility of percentage nor direct suggestions relative to the improvement of the teaching of percentage but that it contributes (1) "data on difficulties in percentage and (2) data on the nature of error, which will add to our knowledge of what error really is psychologically".

Data for the study were taken from the performance, on the Compass Diagnostic Test XIV, of 215 pupils in grade seven in typical classrooms in eight mid-western towns.

The 113 items of the test are presented with figures showing the per cent of failure by children in the lowest and in the highest quarters in the distribution of total test scores.

From the tabulation the author finds that the teaching of percentage is failing in these aspects: (1) recognizing quantities over 100% of a given quantity, (2) recognizing the correct solution of examples using the third case of percentage, (3) solving by inspection easy examples

using the second case, (4) solving by inspection easy examples using the third case, (5) written work using the second case, and (6) written work using the third case.

From an analysis of the specific difficulties in the 35 test items the author concludes (1) that our present methodology of teaching percentage fails to give a clear appreciation of the ideas of percentage, (2) that transmuting a percentage example into a fraction or whole number example is an interference habit which should receive definite attention, (3) that many errors are instances of correct connections wrongly attached.

From a correlation of the scores on percentage test with scores on other tests the author finds that bright pupils do not make one list of errors and dull pupils another list but bright and dull make the errors in about the same relative proportion only the dull make them more frequently.

LIBRARY SUMMARY

Clapp, Frank L., Chase, Wayland J., and Merriman, Curtis
A Study of the Effectiveness of Two Kinds of Teaching Material.
Introduction to Education, p.420-424. Ginn, 1929.

This section of Introduction to Education reports a study made by Marguerite Wilker.

Pupils from 23 rooms, divided into two groups of equal learning ability and equal arithmetic ability, having the same teacher, and the same allotment, were given drill on Case I and Case II in percentage. The experimental group

was given 60% as many Case I and Case II problems as the textbook but the character and arrangement were such as to force the attention of the pupils to the kind of problems which they were solving.

Data are presented in two tables.

The author finds that the pupils using the experimental material did better on the final tests consisting of thirty problems than did the control group.

Mahoney, Nano
Instruction in Percentage.
Elementary English Review 6:161-163. June 1929.

This article reports an analysis from a larger study made by the author.

Unit skills are listed for these phases of percentage:

A. Underlying basic notion.
B. Ideas of whole and parts.
C. Changing decimals to per cents.
D. Changing common fractions to per cents.
E. Changing per cents to common fractions.
F. Changing per cents to decimals.
G. Expressing ratio as per cent.
H. Case I in percentage.
I. Case II in percentage.
J. Case III in percentage.

DISCUSSION

Garlick, Sayrs A.

From Decimal Point to Percentage Symbol.

Chicago Schools Journal 7:256-263. March 1925.

This article discusses drill in reading and writing of whole numbers and of decimal fractions, drill in prefixing and annexing zeros, drill in dividing and multiplying by moving the decimal point, per cent and decimal equivalents, division of decimals with emphasis on placing the point before computing the answer, introducing the per cent symbol. Pattern sentences for six types of percentage problems are illustrated.

SUMMARY OF CHAPTER ELEVEN

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Kircher, H.W. 1926	Error study based on a test of ten items.	133
	Edwards, A. 1930	Error study based on 113 test items.	215
Library Summaries	Clapp, F.L. Chase, W.J. and Merriman, C. 1929	Summary of a study by Miss Wilker on problem solving in CaseI and Case II in percentage.	23 rooms
	Mahoney, N. 1929	Summary of a larger study reporting unit skills in percentage.	
Discussion	Garlick, S.A. 1925	General suggestions for teaching percentage.	

CONCLUSIONS

The most comprehensive study of percentage, a major research no doubt, that made by Miss Wilker, was unavailable.

Only the summary was obtainable.

The results of the error studies by Kircher and by Edwards agree in showing that the present teaching of percentage fails to give adequate concepts of percentage or satisfactory ability in computation.

The general discussion by Garlick completes the few studies of percentage.

Chapter Twelve

DENOMINATE NUMBERS

MINOR RESEARCH

Springer, Isidore

Teaching Denominate Numbers.

Journal of Educational Psychology 6:630-632. December 1915.

100 children in grade six A were divided into two groups on the basis of an interest test of seven problems and of language tests.

Cubic measure was treated in one class by the traditional method of having the class repeat the table individually, in unison, in written form on the blackboard, and on paper. The other group solved five or six problems a day referring to a mimeographed sheet for the units of measure as needed. Ten minutes each day for three days was devoted to this practice in each class.

On a similar test given on the fourth day the traditional class obtained 58%; the class taught by the applied method obtained 64%.

The next week the classes alternated methods practicing 5½ yards equal 1 rod and 320 rods equal 1 mile. In a test the class taught by the traditional method made 62%; the class taught by the applied method made 68%.

From these results the author concludes that the applied method is superior to the traditional method.

Recently interest has been shown in curriculum phases of denominate numbers. In the February 1932 issue

of Education are discussions by Wilson "Why Non-metric Measures in Metric Europe", page 319, and by Louth "Units of Measurement in Industry", page 315. A thesis "The Metric Versus the English System of Measurement in the Teaching of Arithmetic" by Holt prepared in the University of California, argues for the adoption of the Metric System. The argument loses force in view of its basis upon opinions of teachers and examination of a limited number of textbooks.

The study by Springer was the only one, discovered by the writer, with any bearing on the procedure for teaching denominate numbers. This unfortunately involved little used units of measure. Further studies are needed to show the gains in accuracy and in time in teaching relations between units by use instead of by tables, unless the teaching of denominate number tables has already been abandoned by a majority of teachers.

Chapter Thirteen

CHECKING COMPUTATION

MINOR RESEARCH

Clark, John R. and Vincent, Leona E.
A Study of the Effect of Checking upon Accuracy in Addition.
Mathematics Teacher 19:65-71. February 1926.

This article reports a study involving 87 children in grades five and six who were given an initial test of twenty-five single column additions. On the basis of mental ages, I.Q.s, and initial test scores they were divided into two groups. After twenty days practice in adding the initial test was repeated.

The index combining speed and accuracy was taken to equal $\frac{100R^2}{A}$ when R equals the number right, and A equals the number attempted.

The type of check used by the checking group was adding the column in the opposite direction.

Data are presented in three tables.

The conclusions of the authors are that:

1. The practice of checking resulted in greater efficiency in adding. The median of the 22 daily median index scores was 1225 for the checking group and 1050 for the non-checking group.
2. The number of additions attempted in a given time favored the non-checking group. The median of the daily median ttempts was 14.2 for the checking group and 14.9 for the

non-checking group.

3. The number of correct additions (not the number of separate columns) favored checking. The median of the daily median rights was for the checking group 13.1 and for the non-checking group 12.6.

4. A random sampling of two thousand column additions showed 77.9 in accuracy for the non-checking group and 90.1 for the checking group.

5. The increase in accuracy may justify the substitution of this method of checking for the methods now generally in use in drill.

6. Checking transferred to complex addition but not to multiplication. Without checking there was a transfer to multiplication. Data on this point justify mention only.

DISCUSSION

Herr, Ross

Checking in Mathematics.

Chicago Schools Journal 13:17-20. September 1930.

The purposes set forth for the checking of computation by children are: (1) to find their own errors - to develop and maintain a standard of accuracy by themselves, (2) to develop the idea of relationship between processes, (3) to give a real and practical application for each process which is used as a check - also to give practice, (4) for the social utility of checking.

Checking methods suggested for addition include the reverse order which the author considers best, dividing into

two parts which reduces the attention span, and casting out nines which is not generally used. For subtraction adding remainder and subtrahend is suggested. The author advocates checking by addition when beginning multiplication, by multiplication when the multiplier and multiplicand contain the same number of digits, and by division when the multiplier contains one figure and the multiplicand contains two or more digits.

Checking should not be used with whole numbers alone but with common and decimal fractions, according to the author, who further states that checking should be begun after the processes are fairly well formed.

Russell, A. H.
Checking Calculations.
Journal of Education (An English publication) 63:384. June 1931.

This article discusses briefly casting out nines, casting out elevens, applying it to money (English), and the checking of computation in finding areas, in work with compound numbers and with fractions.

SUMMARY OF CHAPTER THIRTEEN

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Clark, J.R. and Vincent, L.E. 1926	Experiment to determine result of checking column addition by adding in the opposite direction.	87
Discussion	Herr, R. 1930	Discussion of reasons for checking, best checking methods, and use of checking with common and decimal fractions as well as with integers.	
	Russell, A.H. 1931	Brief discussion of casting out nines and elevens, and of checking computations in finding areas, in work with compound numbers and with fractions.	

CONCLUSIONS

Only one experiment and that minor in nature has been reported in connection with checking computation. This study by Clark and Vincent finds greater accuracy in column addition as a result of checking by adding in the opposite direction.

Two discussions by Herr and by Russell have a value in bringing the matter of checking to the attention of the teachers. Perhaps the little interest shown by investigators in this problem is an indication of the lack of attention given to checking by the teachers of arithmetic. Not only is there need for investigation in regard to the possible efficiency gained by checking types of computation other than addition but also a need for investigations to determine the relative efficiency of possible checking methods.

Chapter Fourteen

MOTIVATION

MAJOR RESEARCH

Hoover, J. H.

Motivated Drill Work in Third Grade Arithmetic and Silent Reading.

Journal of Educational Research 4:200-211. October 1921.

This article reports a study involving 1139 third grade children from thirty rooms, 571 of whom were in non-drill sections and 568 of whom were in drill sections. Arithmetic cards 2" by 1", with numbers written on them according to the arrangement in the case of dominoes, were used by the children playing in pairs in a game modified from dominoes. The children were divided into drill and non-drill groups according to the judgment of the superintendent. The drill groups used the materials described above for 10 minutes a day on three days each week from December to April. The same daily assignments and the same amount of time were used in both classes with no special directions to the teachers using the drill materials other than to emphasize both speed and accuracy. Results were based on the Cleveland Survey Test given at the beginning and at the end of the experiment. The non-drill section changed from a median of 10.9 to a median of 15.1 examples correctly solved in thirty seconds, a gain of 4.2 examples. The drill group changed from a median of 9.7 to a median of

18.5 examples correctly solved in thirty seconds, a gain of 8.8 examples.

Two tables present the data for arithmetic.

Comment: Equating of groups on the judgment of the superintendent is not in keeping with present more scientific plans. The use of the game by the children in pairs is a gain over class drill which offers few repetitions per pupil per minute.

Wilson, Guy M.
The Motivation of Arithmetic.
Bulletin 1925 #43 of the Bureau of Education of the Department of the Interior. Government Printing Office 1926.

The United States Bureau of Education in 1924 collected 5000 or more replies from teachers throughout the country in response to the request for instances or illustrations of motivated work in arithmetic. These illustrations are classified under five headings: (1) actual life situations, (2) idealized situations, (3) real games, (4) game devices, and (5) devices. Nineteen real and twenty-nine idealized situations are described, the grade in which the situation occurred being noted. Six real games, several game devices for each fundamental process, and a collection of devices not based on games completes the bulletin.

Comment: This bulletin should be known by teachers of all grades not only for the theory underlying motivation to be found in the introduction, but for the distinction between games and devices which should lead to an analysis of many so-called "games".

Panlasigui, Isidoro and Knight, F. B.
The Effect of Awareness of Success or Failure.
Twenty-ninth Yearbook of the National Society for the
Study of Education : 611-619. 1930.

Two fourth grades were involved in this study, one of which used standards and graphs of pupils' own scores as motivation. The drill material contained fifteen examples in mixed fundamentals.

Four tables are included.

The conclusions of the authors are (1) that this study doesn't pretend to throw light on the advantages and disadvantages of motivation using hypothetical or arbitrary standards such that ratings are mixtures of progress and inequalities of difficulty in drill material, (2) that this study applies to a method of motivation wherein ratings furnish valid measures, (3) that the greater the success the greater is the motivation with no significant disadvantage in the case of the slow, (4) that this plan of motivation is equally good for boys and for girls, and (5) this plan had no effect on the variability within the group.

MINOR RESEARCH

Matthews, Pauline E.
Report of Experiment on the Value of Plays and Games in
al
Arithmetic/Drill.
Elementary School Teacher 14:319-324. March 1914.

This experiment involved 26 children in a fourth grade of a training school and lasted for nine weeks. Three motivating activities, the baseball game, the road race, and

the department store are described with the preparation, the scheme of the activity, and the comparison of the results with those obtained through formal drill, being given attention. More examples were solved in the playing of the games, in itself a gain but the author points out a greater gain - the joy which was carried over from the games to the formal drill.

Reavis, W. C.
The Social Motive in the Teaching of Arithmetic.
Elementary School Journal 18:264-267. December 1917.

Following the teaching of stocks and bonds, in an eighth grade, through the organization of a mock bank, a class of 22 was given a test on stocks consisting of ten problems taken from a text-book. The same test was given in September after an interval of ninety days of school and eighty days of vacation.

The following table occurs on page 265 of this reference:

Problems attempted	Problems right	Pupils in first test	Pupils in second test
10	10	15	13
10	9	0	1
9	9	2	4
9	8	1	0
8	8	0	2
7	7	3	1

The above results show according to the author the

persistency of learning when the method provided group activity in which the social motive was pronounced.

Steinway, Louise S.
An Experiment in Games Involving a Knowledge of Number.
Teachers College Record 19:43-53. January 1918.

The purposes of this study, as stated on page 43, were to see (1) "to what extent games involving number are of interest to children; (2) what games afforded possibilities in the matter of points of departure for formal work in arithmetic; and (3) to what extent a knowledge of number could be gained through games."

21 first grade children were involved in this study. A teacher came to them for an hour every day for six weeks to teach them to play games allowing them freedom in joining or leaving the play group. A second teacher took detailed notes regarding the activities of the children.

The games played were lotto (homemade and commercial), waterwheel, roly poly, parchesi, and bean bag. For homemade lotto the average interest span was greatest, being 25 minutes in comparison with an attention span of 14 minutes in waterwheel. During one week 14 children played parchesi while six played bean bag. Homemade lotto was played twenty-eight times by all of the children while bean bag was played eight times. The groups playing lotto averaged five in number while the groups playing bean bag averaged but two.

One table and six graphs show results of this study.

Tentative conclusions point to an educational value

in lotto which proved interesting as measured by attention span. A minus correlation was found between the average interest span and the number of times a game was played. Games led to a need for a knowledge of arithmetic. Much willingness was shown in helping one another. In comparison with a class devoting part of its class period to formal work, test results show an improvement of 3.4 in the game group over 1.9 for the game and formal work group. Children seemed more interested in the activity itself than in winning the game which suggests that scores may not be the best point of departure for formal work.

Comment: The author recognises the limitation in the size of the group and in the uncontrolled factors.

DISCUSSION

Millis, Jas. F.

Appeal to the Play Instinct in Teaching Arithmetic.

The Educational Bimonthly. (The Francis W. Parker School, Chicago) June 1912.

The author suggests the discarding of adult and puzzle problems and the substitution of childrens' experience. He advocates the use of play (1) to motivate drill and (2) to make commercial activities concrete through dramatization. Games are classified according to those (1) purely of activity such as Pussy Wants a Corner, and Simon Says Thumbs Up, (2) imaginative games illustrated by Playing Fireman, Going to Boston, and Playing Store, and (3) scoring games such as Bean Bag, Ring Toss, Ten Pins, Jumping Jack,

Tumble In, Crazy Traveler, and Fish Pond. Dramatizing commercial activities is also illustrated.

Millis, James F.

The Social Motive in Arithmetic.

Kansas School Magazine. September 1912.

This article suggests that we must provide activities for the pupils in which the controlling motive is the desire to do or make something for others - the motive of social service. These social activities are to provide motivation and application for drill, not to replace drill. Illustrations of this idea are caring for chickens, and construction by the eighth grade of a house for the little children. Business dramatizations as in the above article are illustrated.

SUMMARY OF CHAPTER FOURTEEN

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Hoover, J.H. 1921	Comparison of results obtained from unmotivated drill with those obtained by using a modified domino game.	1139
	Wilson, G.M. 1926	Collection and classification of illustrations of motivation in arithmetic contributed by teachers in the schools. This classification is preceded by a review of the history and underlying psychology of the motivation movement.	5000
	Panlasigui, I. and Knight, F.B. 1930	Experiment using standards and graphs of pupils own scores as motivation.	Two fourth grades

Minor Research	Matthews, P.E. 1914	Use of baseball game, road race and store as motivating activities in a fourth grade.	26
	Reavis, W.C. 1917	Retention of knowledge of stocks and bonds taught through organization of mock bank.	22
	Steinway, L.S. 1918	Interest in and arithmetic outcomes from use in first grade of games involving number.	21
Discussion	Millis, J.F. 1912	Suggestions for use of games and dramatizations as motivating activities.	
	Millis, J.F. 1912	Argument for activities in which the motive is a desire to do or to make something for others.	

CONCLUSIONS

These few studies may seem a poor showing in a field receiving as much attention as that given motivation. With the exception of the studies noted other contributions were descriptions, often brief, of the use of one or several motivating activities. On the whole these articles seemed too far removed from real research to warrant inclusion in this chapter. A bibliography of such articles would be of service to the beginning teacher.

Chapter Fifteen

DRILL

MAJOR RESEARCH

Kirby, Thomas J.
Practice in the Case of School Children.
Teachers College, Columbia University, Contribution to Education #58. 1913.

This study involved 1350 children in thirty-nine classes in third and fourth grades.

The addition material was Thorndike's addition sheet which contained 48 columns of 10 addends each of one-place numbers. The division material included the facts from 20 ÷ 3 to 89 ÷ 9.

75 minutes were devoted to this practice, the initial and final drill periods being 15 minutes each, the remaining 45 minutes being distributed differently for each group. In addition Group I had two periods of 22½ minutes, Group 2 had three 15 minute periods, Group 3 had eight periods seven of which were 6 minutes in length and one of 3 minutes, and Group 4 had twenty-two periods of which twenty-one were 2 minutes in length and one was 3 minutes. In division the total practice time was 60 minutes with the initial and final periods 10 minutes each. The remaining 40 minutes were divided into two 20 minute, four 10 minute, and twenty 2 minute periods. In scoring addition the score was the number of columns right and in division the score was the

number of combinations right.

Sauble, Irene
The Effect of Time Allotment Upon Achievement in Arithmetic in the Second Grade.
Master's Thesis, University of California. 1930.

This thesis reviews the studies of Rice, Stone, Payne, Holmes, Ayer, Mann, Hagen, and Chapter 3 of the 29th Yearbook by West, Greene and Brownell.

The study involved ten classes of 40 pupils each in the low second grade and twenty classes in the high second, the children being chosen on the basis of average intelligence, nationality, race, and social and economic conditions of the homes. Teachers in each school chose the two groups by putting the same number of A's, B's, etc., in each group. The Detroit First Grade Intelligence Test was used. The experiment lasted for twenty weeks.

The control group was taught for the regulation 20 minutes, the experimental group for 30 minutes daily. There was no home work or outside coaching. No pre-test was given because no time had been allotted to arithmetic below second grade and it was felt that the number of items on a formal written test to which two B pupils could respond would be an inadequate basis for matching and pairing. Using intelligence test scores, the pupils were matched as to potential ability. When the experiment was repeated in high second grade pre-tests in the addition and subtraction facts were given.

The same number of facts were developed and practiced the experimental group spending the extra time on intensive drill. Developmental material, simple word problems, and drill were provided by experimental editions of texts in which the author was collaborating.

Achievement was measured by scores on four tests in the work-book and on two final tests on 100 addition and 100 subtraction facts. The Detroit Computation Test given in low third was also used. The Standard Deviation Unit was used as a unit of measure of difference.

Data are presented in eight tables and eleven figures.

The author's conclusions are:

1. Nearly all pupils in low second gained some from the additional time but pupils in high second grade made no gain in achievement as a result of the additional time.
2. The amount of gain in low second grade was not proportionate to the additional time used.
3. Pupils in low second grade of below average, average, and above average intelligence appeared to profit equally from the longer period.
4. Pupils in low second grade of average or above average for their grades profited while very young children in the six-year group made no gains.
5. Both colored and American children in the low second grade profited but foreign children made no gain.
6. The additional time affected addition and subtraction in the same manner.

The author thinks that a repetition of the experiment with the extra time used as a separate drill period instead of a part of the regular period would be worth while. Fatigue may have influenced the results although the teachers were told to vary the procedure.

A bibliography of 19 references is followed by an appendix containing samples of instructional material, tests, sample directions for teachers, forms for recording data, explanation of computations, tables 10 to 19 showing more complete findings.

MINOR RESEARCH

Brown, J. C.

An Investigation of the Value of Drill Work in the Fundamental Operations of Arithmetic.

Journal of Educational Psychology 2:81-88. February 1911.

51 children in grades six, seven, and eight were given 5-minute drill periods.

A comparison of sections having drill with sections not having drill but otherwise the same instruction showed that the benefit included increased efficiency in reasoning; that improvement was still evident after a twelve weeks vacation; that the greatest gain was in grade six followed by grades seven and eight.

Data are presented in five tables.

Brown, J. C.
An Investigation of the Value of Drill Work in the Fundamental Operations of Arithmetic.
Journal of Educational Psychology 3:485-492. November 1912.

Two groups totaling 222 children from three schools were given an initial test, 5-minute drills on the four fundamentals for twenty periods and then a final test.

The author finds that the drill classes made from 2 1/5 to 3 1/10 times as much improvement as the non-drill classes.

Brown, J. C.
An Investigation of the Value of Drill Work in the Fundamental Operations of Arithmetic.
Journal of Educational Psychology 3:561-570. December 1912.

This article contains comparisons of the 19 poorest pupils in the non-drill group with the average in the non-drill group and with the 19 best in the non-drill group.

Phillips, F. M.
Value of Daily Drill in Arithmetic.
Journal of Educational Psychology 4:159-163. March 1913.

The groups were equated on the basis of the Stone tests on fundamentals and reasoning. One group was given 10 minutes daily drill on the fundamentals both oral and written and on mental problems. The other group was given no drill work but carried its regular work. The investigation continued for two school months.

There were found to be substantial increases in the

abilities of both groups. The drill group made 15% more improvement in fundamentals and 50% more in reasoning than the non-drill group. The median increase was from 12.9% to 63.2%. The average deviation was greater in the second test, less deviation being found in the non-drill groups. The coefficient of variability was less in the second test than the first. Girls made better records in fundamentals and boys in reasoning.

Mayberry, L. W.
Individualizing Problems for Children.
Elementary School Journal 18:133-137. October 1917.

Tests were given to second and third grade children in October after which five second and five third grade teachers volunteered to check their pupils on individual failures each week. Following the check on errors drill to correct individual errors was administered.

Six tables show the results.

The percentage of gain made by the three "checked" classes in grade two A was 41% compared with 18% gain made by the six "not checked" classes in that grade. In grade two B the gain made by four "checked" classes was 60% and that made by eight "not checked" classes was 36%. In grade three A the "checked" and "not checked" rooms gained respectively 67% and 3%. The grade three B "checked" and "not checked" rooms gained respectively 106% and 23%.

Pyle, W. H.
Economical Learning.
Journal of Educational Psychology 4:148-158. March 1913.

This article is primarily of interest psychologically. Addition is mentioned as one of several types of material.

Third grade children were equated on the basis of an initial test of 10 minutes. One group had 10 minutes practice in the morning for two weeks, the other group had a 10-minute practice period in the morning and in the afternoon for one week.

Results favor one practice period a day.

Thorndike, E. L.
On the Relation Between Speed and Accuracy in Addition.
Journal of Educational Psychology 5:537, November 1914.

Of 671 students the best 65 averaged 150 additions per hundred seconds, the worst 20 averaged only ¼ as many. The 65 made 7 errors per thousand additions and the 20 made 17½ errors per thousand additions.

The table which shows the number of additions per hundred seconds and the number of errors per thousand additions for both slow and fast workers leads to the conclusion that the individual who is quick in adding is also more accurate than one who is slow.

Garfinkel, Maurice A.
The Effect of the Summer Vacation On Ability in the Fundamentals in Arithmetic.
Journal of Educational Psychology 10:44-48. January 1919.

During the vacation the play group did whatever desired, the work group was employed, and the study group attended summer school. The Courtis Fundamentals B Forms 1 and 2 were given in June and September to 861 children of whom one-third were girls.

Results show a general decrease in speed from June to September except in addition. The work group did better than the study group and as well as the play group.

There was a decrease in accuracy in all groups, the highest median in September being lower than the lowest in June, representing a loss of more than two years work.

The decrease in addition was significant pointing out that not enough conscious attention is given to the formation of habits.

The study group had the greatest loss in speed except in addition where it made the greatest gain. In accuracy the work group lost less than the play group. The study group made greater gain in addition and subtraction and less gain in multiplication and division.

Reed, H. B.
Distributed Practice in Addition.
Journal of Educational Psychology 15:248-249. April 1924.

This study involved 203 first and second year college

students who practiced for an hour adding five two-place numbers with no zeros. Group 1, containing 60 students, added for an hour continuously; group 2 containing 50 students added for 20 minutes a day for three days; group 3 containing 51 students added 10 minutes a day for six days; and group 4 added for 10 minutes twice a week for three weeks.

Results show that the group adding for 20 minutes a day made the most improvement but not much more than was made by the groups adding for 10 minutes for six days or 10 minutes twice a week for three weeks. The three above groups made more gain than the group adding for a continuous hour.

Myers, Garry C.
Persistence of Errors in Arithmetic.
Journal of Educational Research 10:19-28. June 1924.

The responses of four children to two facts, each over a period of time, and the responses of several children to examples were recorded.

The author concludes that a mistake is immeasurably serious, that we can never be certain that the same mistake will not be made in exactly the same way again, and that a wrong answer is as definite as a right answer.

The author suggests the reasons for errors to be (1) too much reasoning in teaching the facts and in correcting errors, (2) counting, (3) too much trying to get speed leading the child to guess.

To avoid the counting habit the author suggests

that we (1) substitute accuracy for speed, (2) eliminate devices leading to counting, (3) eliminate telling the child to "stop and think", and (4) master old facts before taking on new.

The author further tells us that drill too often means testing and testing on what is learned wrong, whereas drill should be overlearning correct bonds as repetitions of correct answers with enough of this overlearning in early stages to make correct answers permanent.

Newcomb, R. S.
Securing the Maximum Amount of Work from Every Pupil.
Elementary School Journal 25:376-379. January 1925.

An experiment carried on during six weeks in grades seven and eight consisted of making a definite assignment in a regular text and an optional assignment in a supplementary text. Although no extra credit was offered the eighth grade children solved 2997 problems, an average of 4.3 extra problems per pupil each day. Seventh grade children solved 804 extra problems, an average of 1.9 additional problems per pupil each day. Results on tests given before and after this six week period show a gain of 12% in speed and 14.3% in accuracy in the eighth grade and a gain of 12% in speed and 2.8% in accuracy in the seventh grade.

Clark, John R. and Vincent, Leona E.
A Study of Variability in Arithmetic.
Journal of Educational Psychology 16:267-274. April 1925.

87 fifth and sixth grade children were given drill for 4 minutes a day on twenty-two successive days. The drill material consisted of sets of single column additions of nine digits, no digit less than four being repeated.

Results are shown in six tables.

The conclusions are:

1. The medians for the group for the number of attempts varied by two examples or more.

2. The median for the number of correct additions varied 1.3 examples.

3. Individuals varied from day to day by 2.1 columns or more in speed and by 2.5 columns or more in the number right. There was a tendency for the amount of variability to increase with intelligence.

4. The medians of the numbers of examples correct on the Woody-McCall Mixed Fundamentals varied not more than 1.1 in any of the three school grades tested. The medians of the numbers of examples correct at the end of the first five minutes were unstable.

Ludeman, W.W.
Speed and Scholarship Versus Arithmetical Accuracy.
School Science and Mathematics 25:522-524. May 1925.

44 college sophomores were timed with a stop-watch while doing 50 addition examples. Errors were noted.

The correlation between speed and accuracy was .42, the correlation between the students' grades and accuracy was -.069.

The author concludes that accuracy does not depend on speed, that accuracy does not depend on scholarship, and that speed does not depend on scholarship.

Messick, A. I.
Effect of Certain Types of Speed Drills in Arithmetic.
Mathematics Teacher 19:104-109. February 1926.

Four sections each of fourth and fifth grades totaling 272 children were divided so that half of the sections formed the speed group, the other half forming the control or slow group. Each group was drilled by the writer for 4 minutes a day for twenty days, the drill consisting of eight examples containing nine three-place numbers compiled from the Courtis drill material. Courtis Series B I and II were given at the beginning and at the end of the study. The speed group was directed to add rapidly, the control group to add slowly.

Three tables present the data.

The conclusions are as follows:

1. The accuracy group exceeded the speed group by 2.6% in speed and by 32.5% in accuracy.
2. 4.75% more pupils gained in speed from accuracy drills than from speed drills. 7.4% more pupils drilled for speed lost in speed than did those drilled in accuracy. 2.7% more pupils drilled for accuracy remained the same in speed than did those drilled for speed. In the group drilled for accuracy 2.9% more pupils gained in accuracy than did those drilled for speed. 3.9% more pupils of the speed group lost in

accuracy than did those in the accuracy group. In the accuracy group 1% more pupils remained the same in accuracy than did those in the speed group.

Brueckner, L. J.
Certain Arithmetic Abilities of Second Grade Pupils.
Elementary School Journal 27: 433-443. February 1927.

Pupils in grades two B and two A in eight schools in Minneapolis were given three tests, the test dates being June 17, September 9, and September 23. Grades two B were given 64 addition combinations with sums ten or less and 64 subtraction combinations with minuends ten or less. Grades two A were given the 100 addition and the 100 subtraction combinations. A few summer school children were recorded separately.

Results show that for grades two B there was almost no loss in addition, the boys and girls doing the same kind of work. The summer school children lost 2.7 combinations between June 17 and September 9 but had gained it back by September 23. In subtraction a loss of 2.4 combinations resulted from June 17 to September 9, the boys losing little more than the girls. The summer school children lost and were still below in the September 23 test. In grade two A the medians were

	June 17	September 9	September 23
Addition	98.3	95.8	96.9
Subtraction	96.0	89.3	92.6
Summer School	79.0	68.3	67.5

In grades two B the eight-year olds lost more than the seven-year olds from June 17 to September 9 but the groups were the

same on September 23. In grades two A there was little difference in addition but in subtraction the eight-year olds were below on every test.

Knight, F. B.
The Superiority of Distributed Practice in Drill in Arithmetic.
Journal of Educational Research 15:157-165. March 1927.

This study is preceded by a discussion of drill specifications. The study itself was carried on by Dr. Luse. 600 children in fifth grades were divided into two groups on the basis of general arithmetic ability. Each group was given fifty consecutive 15-minute drill periods, one group using the Knight-Studebaker-Ruch work-books, the other using more material not so distributed. A series of tests included (1) combinations on which both groups had had much practice, (2) combinations on which one group had had liberal practice, the other group not so much, and (3) combinations on which one group had had relatively little practice.

Two tables on distribution are given but for findings reference is made to the study by Luse, viz.:

1. The distributed drill group gained from 20-54% in the number of combinations tried and from 31-85% in the number of examples correct. The haphazard drill group gained from 11-40% in the number of combinations tried and from 13-61% in the number of combinations correct.
2. The gain of the distributed drill group over the haphazard drill group was for addition 18%, for subtraction 19%, for multiplication 35%, and for division 24%.

3. The same relative differences in the distributed and non-distributed drill groups held for all levels.
4. The permanent gain was greater for the distributed drill group as measured by the loss during the summer vacation.

Taylor, Roy E.
Self-Competition Versus Group or Individual Competition in Learning the Four Fundamentals of Arithmetic, with Special Emphasis Given to Speed and Accuracy.
Master's Thesis. University of Kansas. June 1927.

The introduction of this thesis calls to attention the need for evaluating teaching methods instead of accepting the method most advertised. Summaries are given of studies by Shenk, Courtis, School Efficiency Monograph, Brown, Osburn, Childs, Uhls, Batson and Combellick, and Knight.

This study involved 129 children in grades three to seven, in the school year of 1925-26, and 105 children in grades four to eight, in the school year of 1926-27. The initial test, Courtis Research Series B Form 2, was followed by six weeks of work motivated by group-competition, then a test, then six weeks of work motivated by self-competition and a final test. During the self-competition period the Courtis Practice Sets were used. During the group-competition the work was carried on by the teacher dictating problems in the four fundamentals and allowing the pupils to compete with one or another individually or in groups. The idea stressed to motivate this type of drill was, work

to excel the other pupil or pupils, competition not being limited to certain members of the class, but every member competing with all. No records at all were kept of individual or class scores.

Data are presented in sixteen tables.

Summing up the arithmetical operations for each grade it was found that seven relative differences favored self-competition and eleven favored group-competition. In general then, the author finds that individual or group and not self-competition is favored. Neither excelled exclusively in any one grade or in all of the operations.

Weerdt, Esther Hurley de
The Permanence of Improvement of Fifth Grade School
Children in Certain Mental Functions.
Journal of Educational Research 16:127-131. September 1927.

49 children in a fifth grade in New Haven of an average age of ten years were given seven different tests on eleven consecutive days, 45 records being made. The tests were substitution 44 minutes, addition 55 minutes, reading 16 minutes, cancellation 33 minutes, same-opposite 13 minutes, multiplication 55 minutes, multiplication by substitution 44 minutes. Four months later the same tests were given again.

Results for addition on the first day of the re-test were the same as on the fifth practice day and on the second day of re-test were the same as on the ninth practice day. Results for multiplication on the first day of re-test were

the same as on the eleventh day of practice and on the second day of re-test were better than on the eleventh practice day. Results for multiplication by substitution were the same on the first day of re-test as on the eighth practice day and on the second day of re-test were better than on the eleventh day of practice.

The author concludes that in view of the relatively small amount of time spent in specific drill in the seven functions tested, the amount of improvement retained through four months of non-specific drill is significant.

Long, Wilbur E.
Pupil Attention in Arithmetic.
University of Pittsburg, School of Education Journal Vol. 3, #2:27-29 and 32-33. November-December 1927.

A survey of classes in arithmetic in Jeannette, Pa. at the beginning and at the end of a course in arithmetic methods was made using the Class Attention Score Sheet of H. C. Morrison.

Loss of pupil attention is found to be due to

a. Wastes due to poor preparation.
  1. Much time is wasted in getting started because materials for work are not ready.
  2. Pupils lose time and interrupt the class because work on the board is not legible.
  3. When questions are not readily identified by number or letter, pupils lose time.
b. Wastes due to poor techniques.

1. The wavering of attention if drill is difficult and the teacher does not frequently refer to the class when difficulties are met.

2. The danger that the game or competition may divert the attention of the class from the arithmetical principles or facts involved to the contest itself, unless care is taken to subordinate the game.

3. The tendency to consider the individual instead of the group, e.g. calling on a single pupil to continue drill until a mistake is made.

4. The difficulty in trying to keep all of class working together throughout the recitation. Other pupils lose time when a pupil is kept on his feet for detailed questioning.

5. The practice of assigning one problem at a time.

6. The practice of having a pupil drone out an explanation of his work at the board.

The solution to this problem of pupil waste lies in (a) understanding the objectives sought, (b) knowledge of methods and devices needed to reach desired goals, and (c) the ability to put across the program in a class room.

Two lessons are illustrated to show 100% attention.

The author lists these devices and methods to conserve attention: (1) Vigorous and wide-awake class control; (2) Neat orderly board preparation; (3) A formulated lesson plan with interest and attention as objectives; (4) Short snappy drills; (5) Mimeographed contracts in supervised study; (6) Games and motivated devices consistent with the age and

interests of the children; (7) Vital material with natural life interest in word problems; (8) Sufficient presentation and explanation to remove major difficulties; (9) Provision of extra work for rapid workers; (10) Individual attack upon problems enough to last a period; (11) Organization to stress participation of all rather than a few.

Hagen, Henry B.
A Study of Practice Periods in Arithmetic Fundamentals.
Chicago Principals' Club - Second Yearbook 93-95. 1927.

Two groups were formed in each of twelve rooms including grades four B to seven A on the basis of the Otis Group and Illinois Intelligence Tests.

In February the Monroe Standardized Arithmetic Fundamental Scale was given to each group.

Group 1 was given drill both in forenoon and afternoon, Group 2 being given drill in the forenoons only.

The Monroe Scale I given May 16 shows Group 1 to have made a median gain of 12.3 examples and Group 2 to have made a median gain of 13.7 examples. The conclusion is that the extra twenty minutes a day was unwarranted.

Myers, Garry C. and Myers, Caroline E.
Finding Mistakes Versus Correct Associations in Simple Number Learning.
Journal of Educational Research 18:25-31. June 1928.

Two groups were chosen alternately from alphabetical order, one-half being given the task of finding mistakes, the other half repeating 20 facts with their answers. The

first test of twenty examples was given to all, the time and answers being noted. After the exercises the same test was given again. The two groups were almost equal in time and accuracy on the initial performance. 100 pairs were matched on the basis of time for the initial performance. 147 children were given the test again with no practice.

Results show the gain as reduction in time and errors to be almost the same for 138 pairs alphabetically as for 10 pairs matched in initial time. One reading of 20 facts and answers was better than the search for errors. The two groups made on the average the same number of errors and practically the same initial time. The error finding group took 50% more time in learning.

Tables are given for time and errors.

The authors conclude that finding errors is wasteful of time.

McMurray, J. Fred
Teaching Arithmetic to Seventh Grade Pupils Orally Versus Teaching it as Written Arithmetic.
Master's Thesis. University of Cincinnati. August 1928.

The units taught during this experiment were finding a per cent of a number, the percentage equation, finding what per cent one number is of another, profit and loss, buying and selling on commission, trade and cash discounts, successive discounts, paying for use of money, estimating results, plane figures, review and drills in fundamentals.

Written arithmetic according to the author includes

all work accomplished with pencils in the hands of the pupils. Work explained from the blackboard is classified as oral. In cases where pupils were working arithmetic at their seats and the teacher passed from seat to seat to assist, all explanations are counted as written method.

During one 50 minute period daily for five days a week for three weeks four groups , W, W', O, and O', used fifty per cent of the daily period working arithmetic as oral arithmetic and fifty per cent working written arithmetic. During the next twelve weeks groups W and W' used seventy-five per cent of the time for written and twenty-five per cent for oral work. Groups O and O' used seventy-five per cent of the time for oral and twenty-five per cent for written arithmetic.

The bases for comparing results were (1) correct principle in solving problems and (2) correct answer in solving problems.

The author concludes that the written method is better in general.

The appendix contains 13 pages relating to the I.Q.s and letter values for 160 children, 8 pages relating to equating and final grades and lessons on trade and cash discounts, successive discounts, interest, and circles .

Comment: There was more detail on the equating of groups than the writer found helpful. She would like to have known more about the division of material into lessons. Do seventh grade units, well taught, lend themselves to an arbitrary division into oral and written work? Do we not rather need an analysis of phases suitable for oral and for written work? Was the

only explanation that given on the mimeographed sheets? What about the part played by the teacher?

Myers, Garry C.
The Price of Speed Pressure in the Learning of Number.
Educational Research Bulletin (Ohio State University) 7:
265-268. September 19, 1928.

21 children in grade one A were given the meaning of addition process and the number concepts to 20. They put together 3 and 4 and counted but made no attempt to memorize. During two months they were given mechanical learning of the number facts using flash cards with answers and without answers. All effort was made to avoid haste, carelessness, and errors. A test on the 63 facts studied resulted in 18 correct papers, three omitting from 1 to 5 facts but no one giving a wrong answer.

Two years later 10 of these 21 pupils were studied. Four individual responses to each card were made, greater speed being suggested each time after the first leisurely response. Results show that the children made more errors on all subsequent trials than on the first, the total number of errors on the five trials respectively being: 14, 36, 39, 37, 43. The average increase in error was from 1.4 in the first to 4.3 in the fifth.

One table giving a record of the wrong answers shows the author's findings - that errors persist. 31% of the errors recurred from 1 to 4 times. Children often responded without seeing the whole combination, answering in terms of

one of the numbers. One pupil made 22 such errors in 28 errors made.

Bruene, Elizabeth
Effect of the Summer Vacation on the Achievement of Pupils in the Fourth, Fifth and Sixth Grades.
Journal of Educational Research 18:309-314. November 1928.

The Stanford Achievement Test A was given in May and test B in September to grades four, five, and six.

Four tables show results for the grades separately, and change in achievement in terms of school years.

The average loss in arithmetic computation was six school months, the average loss in reasoning six tenths of a school month.

The author sees in these results evidence of a need for review. She feels that the borderline children need to go to summer school.

Hamilton, Dwight
An Experiment with an Individual Technique in Sixth Grade Arithmetic.
Journal of Educational Research 18:339-344. November 1928.

16 sixth grade children were taught by the individual technique following a plan similar to the Winnetka plan. Unit sheets were prepared which contained pupil's goal in terms of facts or skills, assignment and practice tests. Compass Diagnostic Tests were used to test mastery. The daily program was revised to replace the recitation with

individual practice and with consultation.

Four tables present the data.

Forty-six units of work were accomplished by the brighter and nine units by the slowest children. Progress, measured by the Stanford Achievement Test, showed a gain by the individual technique of 1.5 grades where the normal gain would be .75 grades.

The author feels that there is value in the individual method which is especially needed in rural schools. He deplores the lack of prepared materials. He suggests that texts must be written to the children, not to the teacher, and that unit materials in texts must be more self-directive.

Nelson, M. J.
How Much Time Is Required in the Fall for Pupils of the Elementary School to Reach Again the Spring Level of Achievement.
Journal of Educational Research 18: 305-308. November 1928.

The Courtis Standard Research Test B was given to grades seven B and five A, the first form in May and the second in September and then at intervals of two weeks for six weeks and again at Christmas.

Grade seven B regained its spring level at the end of four weeks in speed in fundamentals. In problems correctly solved the spring level was reached at the end of six weeks. In grade five A the spring level was reached at the end of six weeks and before the fifteen week test at Christmas.

The author concludes that a review of fundamentals in the fall yields large returns.

Morgan, L. D.
Specific Versus General Drill in the Fundamentals of
Arithmetic in Grade Four.
School Science and Mathematics 29:528-529. May 1929.

This article reports a comparison of results using Lennes Pads and using Economy Remedial Exercise cards with 56 children in grade four. The author finds the rate of gain of Economy Remedial cards over Lennes to be 43.23 per cent.

Comment: This title is misleading since the article really deals with a comparison of two sets of drill materials.

Morgan, L. D.
How Effective Is Specific Training in Preventing Loss
Due to the Summer Vacation?
Journal of Educational Psychology 20:466-471. September 1929.

Two sixth grade classes taught by the same teacher were given tests on May 11. For two weeks following the tests and preceding the close of school one group received specific training in arithmetic fundamentals and in problem solving. Both groups were then given the same tests which were repeated once again in September. Table I shows that the group receiving specific training increased in efficiency 8.10 examples in arithmetic fundamentals while the group not receiving specific drill gained .21 examples. In problem solving the increase for the groups receiving and not receiving specific drill was respectively 2.43 problems and .39 problems. The loss in problem solving between May and September

was greater for the group receiving specific drill but the loss was less in arithmetic fundamentals than for the group not receiving specific drill.

Deakin, Ora Ethel
The Effect of Organized Drill on Success in Arithmetic.
Master's Thesis. University of Colorado. 1929.

This study involved two schools in Texas, two fifth grades in each school totaling 82 and 66 children. Anderson Arithmetic Book II was used by both experimental groups. The Stone-Brown-Hopkins drills were used with the experimental group. The Arithmetic Workbook by Knight, Studebaker and Ruch was used with the control group.

The Los Angeles Diagnostic Tests Form I were given at the beginning of the year as was the Otis Intelligence Test. The Los Angeles Diagnostic Tests Form II were given at the end of the year.

64 children in the control group and 43 in the experimental group were present for all tests and their results furnish the data.

On the Intelligence Tests the median and the mean in the control group were 96.78 and 97.11 respectively, in the experimental group 94.72 and 99.83 respectively.

In the scores on fundamentals the control group gained 15.46 points or 26% of the mean in the first test.

The experimental group gained 30.03 points or 46% of the mean in the first test.

In the scores on reasoning a gain of 1.64 points or

9% was made by the control group and a gain of 2.57 points or 15% was made by the experimental group.

The author concludes that (1) The group using the drills made a per cent of gain approximately twice as great as the per cent of gain made by the group not using drills, (2) In the experimental group the gain in reasoning was twice as great, (3) The per cent of improvement in the fundamentals was three times as great as the gain in reasoning, (4) Since the groups were equated on the basis of initial performance difference in results is due to drills, (5) Drills in fundamentals affected performance but the analysis did not show how.

Comment: There was not enough description of the use of materials. Aren't workbooks by Knight, Studebaker and Ruch drills? Shouldn't this study be called a comparison of two sets of drill materials? What is the effect of the relation of the drill material used to the test contents?

Repp, Austin C.
Mixed Versus Isolated Drill Organization.
Twenty-ninth Yearbook of the National Society for the
Study of Education : 535-549. 1930.

This study involved 631 children and 23 teachers of sixth grade arithmetic. The children were divided approximately equally into two groups, equated on eleven points such as use of text-book, length of period, etc. A pre-test of 139 minutes on twelve operations was given to determine the pupils' status on previously taught material.

The drill material used in this study comprised 26 drill exercises totaling 521 examples in each set, the same set of examples being used in each set of drills, therefore the same bonds. The arrangement of drills was such that the group receiving mixed drills (M) was given small amounts of drill in each process often; the group receiving isolated drills (I) was given extensive drill on each process seldom. One drill was given each week. The same examples were used in the final test as in the pre-test except that as given in the final test they were in mixed order. The author sees no advantage to the mixed group in this. He uses the argument that any test in social situations would favor mixed drill. He further states that no one has shown what type to use, therefore he has used the most socially useful.

The author finds that Group M although not significantly superior to Group I at the beginning of the test was significantly superior at the end of the test. From tables the author finds that pupils using mixed drills were in 100% of the cases superior to pupils using isolated drills. Out of 48 comparisons Group M was superior, in 38 cases significantly. In right answers Group M gained 65%, Group I gained 42%.

Examination of Parts 1, 2, 3, and 4 of the final test show that the group having isolated drills did its best work on the parts containing the larger per cent of denominate numbers, work which was recently practiced, and poorest work on the functions practiced a longer time ago. Again the author feels that the test is not at fault.

The author's conclusions are (1) All pupils profited by drill but the group having mixed drill made 23 per cent greater gain, (2) Pupils making the lowest pre-test scores made the greatest relative gains, (3) The mixed drills were significantly superior to the isolated for maintenance, (4) More gain was made in the functions recently taught, the gain being greater in mixed drills than in isolated, (5) Pupils using mixed drill learned to work more accurately and more rapidly, (6) Weekly drill scores of the group using mixed drill fluctuated less widely and appeared to sustain interest at a higher level for this reason, (7) The division of whole numbers was least affected by drill indicating that that process was most poorly mastered. Comment: As previously indicated the author refuses to see any advantage to group receiving mixed drills in that the final test was in mixed order. What the author's plan amounted to was an investigation of infrequent versus frequent drill.

Burgess, Bliss M.
How Nearly Can We Approach 100% Results in the Fundamentals of Addition and Multiplication in Grades Four and Five?
Master's Thesis. Boston University. 1930.

This study involved 121 children and four teachers in grades four and five and extended from November 1929 to April 1930. Drill given for 5 minutes three times a week made use of the Wilson tests and drill service. Details are given describing the use of this material. Data are fully presented in the fifty-nine tables and fifteen figures.

When the multiplication tests were first given no child had 100%. After correcting addition errors the multiplication test was again given. In grade four the number of errors decreased from 1160 to 289 and the number of children obtaining 100% increased from none to 18. In grade five the number of errors decreased from 1128 to 48 and the number of children obtaining 100% increased from none to 46.

A report of grade and pupil progress is given for each week.

A bibliography containing 21 references is included.

Kramer, Sister Mary Immaculata
Permanence of Improvement and the Distribution of Learning in Addition and Subtraction.
Doctor's Dissertation, Catholic University of America, Washington, D.C. 1931.

The purposes of this study were (1) to determine the relation of length of practice periods to degree of permanence, (2) to determine the effect of concentrated versus distributed practice upon permanence, (3) to measure the influence of a single practice period each day in comparison with a double period of practice, (4) to determine the relation of permanence to the method of drill, i.e. whether written or oral drill is more effective in securing permanent results, (5) to determine for what age permanence is greatest, (6) to measure the effect of the summer vacation upon the permanence of gains.

The study as to effective methods involved 410 chil-

dren and continued from December 2 to December 17. A pre-test was given consisting of 96 examples of four addends each made up of the 100 primary facts equally distributed.

The three methods used were as follows: (1) mimeographed sheets containing 96 examples of four addends each arranged in five forms each of which was used twice, (2) oral work containing the same examples as 1 dictated to the group lined up on the sides of the room, all of the work being oral, (3) work dictated to the children who wrote the answers.

After 50 minutes drill a re-test the same as the pre-test was given. The tables show that the first method brought about the greatest change in speed in all grades. The first method produced the greatest change in accuracy in grade three but the third method produced the greatest accuracy in grades four and five.

The study relating to written versus oral drill involved 2634 children in third, fourth, fifth, and sixth grades in fourteen schools. Fifty-six classes comprised twenty-four addition with four control classes and twenty-four subtraction with four control classes. Of the twenty-four classes sixteen did written work and eight did oral work. These groups were paired on initial ability.

Initial and final tests were composed of 100 examples like add $\begin{array}{r}1\\9\\6\\\underline{8}\end{array}$ and subtract $\begin{array}{r}245\\\underline{132}\end{array}$. Practice drill sheets contained 168 examples like the above. Tables show the number of children, drill method, and time for both addition and subtraction. Drill periods were 3, 6, 9 minutes or double 3 minute periods

per day. Practice continued for two weeks. Three re-tests were given at intervals of two, six, and ten weeks after practice.

The study to determine the relation of the length of practice periods to permanence based on data from the above investigation shows that in the third grade the double 3 minute drill periods gave best results while the 6 minute period gave the poorest results at the six-weeks retention test. On the ten-weeks retention test the single 3 minute period gave best results. In the fourth grade the ten-weeks retention test showed the 3 and double 3 minute periods to be most effective. In the fifth grade the 6 minute period gave best results on both retention tests. In the sixth grade the single and double 3 minute periods were better than the 6 or 9 minute periods. In subtraction 6 minute and 3 minute periods for third and fourth grades respectively and double 3 and 6 respectively for grades five and six proved most efficient.

In discussing concentrated versus distributed drill the author reviews studies by Lorge, Lyons, Lashley, Murphy, Dearborn, Hagen, Cummins, Edwards, Hahn and Thorndike, Kirby, Mould, Treadwell and Washburn, Bergstrom Gordon and Austin. Results of this study favor double practice each day.

In the consideration of the relation of age to permanence the author reviews studies by Wells, Ballard, and Williams. The two week retention test shows permanence related directly to age, the six week retention test shows the same but the ten week retention test shows the reverse. This was

accounted for however by the nature of the work in the interim.

The effect of summer vacation on permanence of gains shows greater loss in subtraction than in addition, the loss being greatest in the third grade. The loss was greatest in the groups using methods 2 and 3.

Summarizing, the author's conclusions are as follows:
1. Short periods of practice, 3 minute or double 3 minute for addition and double 3 or 6 for subtraction, are preferable.
2. The more highly distributed the drill the better the results.
3. Greater and more permanent gains result from written work.
4. The degree of permanence is directly related to the age of the subjects.
5. There is little need for much intensive drill at the beginning of the fall semester. Less drill is needed in lower grades and less is needed for addition than for subtraction.

Osburn, W. J. and Foltz, P. J.
Permanence of Improvement in Fundamentals of Arithmetic.
Educational Research Bulletin (Ohio State University)
10:227-234. April 29, 1931.

Children in ten consolidated, two two-teacher and two one-teacher schools were used in this study. The Wisconsin Inventory test was given in October. In April a test on the hardest 30 facts was given and in September the same test as in April was given.

The percentages of loss were obtained by dividing the number of misses by the number of children.

Four tables show for grades two to six the highest frequency, average frequency, and lowest frequency on these phases: addition and subtraction facts, multiplication and division facts, upper decade addition, and short and long division.

The author finds that the average vacation loss was never more than 3/5 and may be less than 1/12 of the gain. The greatest vacation loss was in those grades in which the material was being taught for the first time. The brighter pupils generally showed large amounts of gain and small amounts of loss. Sometimes small gain resulted because the children were so near mastery on the first test.

Knight, F. B. and Ford, E.
Temporary Lapses in Ability and Error in Arithmetic.
Elementary School Journal 32:111-124. October 1931.

A test using 7 x 9 in different positions such

$$\text{as}\quad \begin{array}{r}2649\\ \underline{837}\end{array}\quad \begin{array}{r}2946\\ \underline{837}\end{array}\quad \begin{array}{r}2649\\ \underline{378}\end{array}\quad \text{was prepared in four forms containing}$$

eight multiplication examples each. This test was given to 200 eighth grade children, the equating of groups consisting in giving every fourth child the same form.

Tables show frequency of error as a function of position in series, two-digit multiplicands in various positions, and multiplicand and multiplier in different positions.

The conclusions are that carrying is not more frequent at the ends and that difficult combinations do not appear more frequently near the ends. The authors call attention to our knowledge that attention flickers, that there

is a decrease of effective output over a long period, and that we make no adjustment for short time fluctuations. As possibilities of correction they suggest that we consider the factor of position in the test, that we use wisdom in handling the child telling the facts and not reproving, and that we consider the rhythmical mental vacations.

Comment: A similar study by Wilson where 100% standards have prevailed shows no validity in the work of Knight and Ford.

LIBRARY SUMMARY

Phillips, Frank M.

Comparison of the Work Done in the Successive Minutes of a Ten Minute Practice Period in the Fundamentals of Arithmetic. Journal of Educational Psychology 7:271-277. May 1916.

This article is an abstract of a thesis. 100 pupils in each of grades four through eight, divided evenly as to sex, took tests on primary facts in addition, subtraction, multiplication, and division. Pupils were directed to mark the point reached at the end of each minute according to signals. It was found that 6-10% more work was done in the first minute of a 10-minute test than in the everage of the later minutes. The poorest scores were most frequently made in the second minute. The work of the first minute was less variable than that of later minutes. There was a uniform increase in the median of the work from the fourth grade to the eighth grade with much overlapping. Errors were less frequent in the first minutes. Sex differences were slight, the girls doing 4% more work than boys, and being about 4% more accurate than boys and about five months younger. There was a rather high correlation between

the work of various minutes. The author concludes that the ability of a pupil in fundamentals may be determined by what he accomplishes in one minute of work.
Comment:Results show comparative general level of a group rather than specific individual ability.

Steel, H. J.
Time Activity Analysis Technique Applied to the Supervision of Arithmetic.
Second Yearbook of the National Conference of Supervisors and Directors of Instruction. Bureau of Publications, Teachers College, Columbia University, 1929.

This is a summary of a thesis "The Time Distribution in the Arithmetic Drill Period" prepared by the author in the University of Minnesota.

From a questionnaire sent to 97 teachers of grades four to six in Minneapolis asking about the distribution of time in arithmetic, data were obtained which are set forth in two tables. Findings indicate emphasis on fundamentals in grade four. Shift of emphasis to fractions and decimals in grades five and six, increasing time on problem solving, isolated not related rpoblems, with progress through the grades, continued time emphasis on fundamental processes with integers in grades five and six.

From 97 teachers in other Minnesota cities and villages these tendencies were discovered: more blackboard and oral drill than any other, few informal tests of achievement, generous use of standardized practice material and timed tests, few project problems but much attention to isolated problems, little graphing of results, little use of student

teachers. The time during study was used by the fourth grade for solving isolated problems, working timed drill, and doing examples to aid in the developing of a new process. In grade five the time was devoted to working examples in fractions and decimals and solving isolated problems. In grade six the time was devoted to solving isolated problems.

DISCUSSION

Wisend, W. F.
Oral Arithmetic.
Education 43:99-103. October 1922.

The author advocates oral drill for at least a third of the class period in upper grades and for a greater part of the period in lower grades. He says problems and examples must come from the pupils not from the teachers. Illustrations of pupil made examples and problems are given.

McAndrew, W.
Accuracy as an Educational End.
Elementary School Journal 26:644-646. May 1926.

This article offers quotations from the Mid-Year Prospectus of Superintendent McAndrew to his division superintendents. He suggests ability levels in requirements for children and attention to individual difficulties. He finds perfection in accuracy being attained, the work being well motivated and not fatiguing.

Laughlin, Butler
The Unit Method of Applied Arithmetic.
Chicago Schools Journal 9:44-51. October 1926.

It is suggested in this article that the material in arithmetic be organized into units selected in accordance with the theory underlying subject matter and the value of the material to society. Morrison's units for the first six grades are listed. The teaching procedure includes exploration, presentation, supervised study, organization, recitation, and drill.

McDade, James E.
Individual Number Drills.
Chicago Schools Journal 9:321-325. May 1927.

These principles are set forth as fundamental to effective drill: (1) The child should answer unhesitatingly from the beginning, (2) When the child doesn't know the answer it should be given to him, (3) Practice should be errorless, (4) Practice should be individual, (5) It should be possible to test oneself and know what mastery has been attained, (6) The method of study should provide definite unvarying procedure, (7) Mastery of old material should precede the presentation of new, (8) The order of presentation should be the order of ascertained difficulty, (9) The amount of practice should be proportioned to its difficulty for the individual child, (10) The teacher should know at all times the attainment of each pupil, (11) The pupil should be accustomed to tests under time limitations with the time

adjusted to each child.

The remainder of the article describes the author's individual number drills.

Myers, G. W.
The Threefold Objective of Drill in Arithmetic.
School Science and Mathematics 28:281-289. March 1928.

The three objectives of public school arithmetic according to this author are (1) intelligent control of number and space procedures, (2) calculatory skill, and (3) problem solving power.

Activities provide (1) motivation, (2) development of concepts, (3) assimilation of ideas, (4) practice to obtain skill, (5) application of new skill, (6) problem tactics and strategy, (7) consolidation of new with old skills, (8) recall of old skills for maintenance.

The three types of drill suggested by the author are; I. Drill for Assimilation - centered on a single skill, furnishing repetitions of the precise skill being assimilated.
II. Drill for Skill Building - furnishing repetitions with attention in close succession allowing no exceptions.
III. Drill for Skill Holding - seeking by periodic recall and re-study to make retention permanent through longer periods.

Benz, H. E.
The Maintenance of Fundamental Arithmetical Skills
in the Junior High School.
Mathematics Teacher 21:326-335. October 1928.

Studies by Morton, by Schorling and by Touton and Terry are reviewed showing inadequate response of junior high school and advanced pupils to arithmetic. The author finds that text-books do not prepare adequately for the maintenance of skills. He suggests that in junior high schools a half hour each week be devoted to a review of fundamental facts and processes.

SUMMARY OF CHAPTER FIFTEEN

Because of the wide range of topics treated in this chapter it seems wise to classify the studies before attempting a summary as to types of studies, nature of material and number of subjects.

VALUE OF DRILL

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Brown, J.C. 1911	Comparison of drill with non-drill groups in grades six, seven, and eight.	51
	Brown, J.C. 1912	Comparison of drill with non-drill groups.	222
	Brown, J.C. 1912	Comparison of 19 poorest pupils in non-drill group with average in non-drill group and with 19 best in non-drill group.	19
	Phillips, F.M. 1913	Comparison of drill and non-drill groups in fundamentals and reasoning.	

DISTRIBUTION OF DRILL PERIODS

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Kirby, T.J. 1913	Comparison of results obtained from 45 minutes of drill in addition, the time being distributed in periods of 22½ minutes, 15 minutes, 6 minutes, and 2 minutes.	1350
	Sauble, I. 1930	Comparison of results obtained from periods of 20 and 30 minutes drill in second grade.	400
Minor Research	Pyle, W. H. 1913	Comparison of results obtained from a ten-minute period in the morning and ten-minute periods in both morning and afternoon in third grade.	203
	Reed, H.B. 1924	Comparison of results obtained from 60 minutes of drill distributed in one continuous period, three twenty-minute periods, and six ten-minute periods with college students.	
	Hagen, H.H. 1927	Comparison of results obtained from twenty minutes of drill in the forenoon and twenty minutes of drill in both the forenoon and afternoon.	12 rooms

EFFECT OF SUMMER VACATION

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Garfinkel, M.A. 1919	Comparison of vacation losses in accuracy and speed in play group, work group, and study group.	861
	Brueckner, L.J. 1927	Vacation losses in addition and subtraction sustained by second grade children as measured on September 9 and September 23.	8 schools

	Bruene, E. 1928	Vacation losses in computation and reasoning in grades four, five, and six.	3 grades
	Nelson, M.J. 1928	Time required in grades five and seven to regain spring level as measured by tests given in September, six weeks later and at Christmas.	
	Morgan, L.D. 1929	Vacation losses in computation and reasoning in a group given specific drill between May 11 and the close of school compared with losses in a group not receiving specific drill.	2 sixth grades
	Osburn, W.J. and Foltz, P.J. 1931	Vacation losses in primary facts in four fundamentals, upper decade addition facts, and long and short division.	14 schools

INDIVIDUALIZING DRILL

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Mayberry, L.W. 1917	Comparison of second and third grades in which teachers checked their pupils on individual failures each week with grades in which this was not done.	
	Newcomb, R.S. 1925	Value to seventh and eighth grade children in solving optional problems in addition to the regular assignment.	2 grades
	Hamilton, D. 1928	An experiment with a plan similar to the Winnetka plan.	16
Discussion	McDade, J.E. 1927	Discussion of principles of effective drill. Description of author's individual number drills.	

ACCURACY AND SPEED IN DRILL

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Thorndike, E.L. 1914	Relation between number of additions per second and number of errors per 1000 additions in the cases of fast and slow workers.	671
	Ludeman, W.W. 1925	Relation of accuracy, speed, and scholarship based on 50 addition examples done by 44 college sophomores.	44
	Messick, A.I. 1926	Comparison of results obtained in groups directed to add rapidly with results in groups directed to add slowly.	272
	Myers, G.C. 1928	Effect of urging greater speed in each successive response to certain repeated number facts.	21
Discussion	McAndrew, W. 1926	Discussion including mention of work for accuracy carried on in the Chicago schools.	

MOTIVATION IN DRILL

Minor Research	Taylor, R.E. 1927	Comparison of results obtained when the motivation emphasized was self-competition with results when individual and group competition were stressed.	129
	Long, W.E. 1927	A survey of arithmetic classes listing wastes due to poor preparation, wastes due to poor techniques, and devices and methods for conserving attention.	

MISCELLANEOUS

Minor Research	Myers, G.C. 1924	Study of responses of four children to two number facts over a period of time showing tendency of errors to persist.	4

Type of Study	Author and Date	Nature of Study	Number of Subjects
	Clark, J.R. and Vincent, E.L. 1925	Study based on single column addition for a period of twenty-two successive days showing variation in results obtained by individuals from day to day.	87
	Knight, F.B. 1927	Comparison of results from distributed and non-distributed drill.	600
	Weerdt, E.H. 1927	Study to determine retention of material given specific drill after four months of non-specific drill.	49
	McMurray, J.F. 1928	Comparison of results from teaching oral and written arithmetic.	4 groups
	Myers, G.C. and Myers, C.E. 1928	Comparison of benefits derived from finding mistakes and from correct associations.	564
	Deakin, O.E. 1929	Comparison of Stone-Brown-Hopkins and Knight-Studebaker-Ruch drill materials.	148
	Morgan, L.D. 1929	Comparison of results obtained from using Lennes pads with Economy Remedial Exercise cards.	56
	Repp, A.C. 1930	Comparison of results obtained from giving small amounts of drill in each process often with results from giving extensive drill on each process seldom.	631
	Burgess, B.M. 1930	Detailed description of use of Wilson tests and drill service for addition and multiplication.	121
	Kramer, Sister M.I. 1931	Report of several studies to determine effect of method, and distribution of drill upon permanence in learning and effect of summer vacation.	
	Knight, F.B. and Ford, E. 1931	Effect of the position of multiplier and multiplicand upon the accuracy of a primary multiplication.	200

Type of Study	Author and Date	Nature of Study	Number of Subjects
Library Summaries	Phillips, F.M. 1916	An abstract of a thesis comparing the work done in the successive minutes of a ten-minute drill period.	500
	Steel, H.J. 1929	An abstract of a thesis determining by means of questionnaire how the class time is distributed in the intermediate grades.	97
Discussion	Wisend, W.F. 1922	Discussion of value and place of oral drill.	
	Laughlin, B. 1926	Discussion of organization of subject matter into units.	
	Myers, G.W. 1928	Discusses need of drill for assimilation, for skill building, and for skill holding.	
	Benz, A.E. 1928	Discussion of need for maintenance drill in junior high school.	

CONCLUSIONS

Interest in the distribution of drill periods dates from Kirby's major research in 1913 and includes Miss Sauble's study of 1930.

Minor researches relating to summer vacations by Garfinkel, Brueckner, Bruene, Nelson, Morgan, and Osburn and Foltz, all, with the exception of the first, having been made within the last six years show this topic to be of current interest.[1] These studies include comparison of losses by

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[1] See Teachers College Contributions to Education #204 by Margaret E. Noonan for effect of summer vacation on all school subjects.

children at work, at play, or at study; time required to regain vacation losses; effect of specific drill just before the close of school as well as measurement of losses due to vacation. In general conclusions point to a fairly considerable loss due to vacation with a need for review at the opening of the new school year.

The value of individualizing drill is treated by Mayberry whose teachers checked individual failures each week; by Newcomb who through optional problem solving provided work for all pupils; by Hamilton who used a plan similar to the Winnetka plan; and by McDade who discusses principles of effective drill particularly his own individual number drills.

Studies of accuracy and speed in drill point, in general, to more effective results ensuing from emphasis on accuracy than from emphasis on speed. Ludeman finds that accuracy doesn't depend on speed or on scholarship and that speed doesn't depend on scholarship while Myers finds serious loss of accuracy accompanying increased speed pressure. Messick finds that drill emphasizing accuracy has a better effect on both speed and accuracy than has drill emphasizing speed. Thorndike in his early study in 1914 finds the person who works rapidly is likely to work accurately, results not entirely in keeping with later studies. McAndrew's discussion refers to work for accuracy in the Chicago schools.

A study by Taylor shows comparison between motivation based on self-competition and that based on individual or group competition. Long points out the loss of attention

due to poor preparation and poor technique.

Other studies in drill include comparisons of distributed and bunched drill by Knight and by Repp; the tendency of errors to persist determined by Myers; variability of individuals from day to day discovered by Clark and Vincent; retention resulting from specific drill studied by de Weerdt; comparison of oral and written arithmetic investigated by McMurray; discussion of oral drill by Wisend; comparison of finding mistakes with correct association as drill devices carried on by Myers; comparisons of certain drill materials on the market made by Deakin and by Morgan; comparison of work done in successive minutes of a ten-minute drill period made by Phillips; determination of manner of distribution of time devoted to drill in the grades summarized by Steel; description of use of Wilson drill service made by Burgess; discussions of purposes and need for drill prepared by G. W. Myers and by Benz; and an investigation by Kramer covering method and distribution of drill, permanence, and effect of summer vacation.

The interest in drill has produced so many studies so varied in nature that the topic has not received justice in its treatment as a chapter in this thesis.

There are a few phases of drill which may well receive the attention of those in a position to make investigations. One of those is the effect of individual drill in comparison with group drill. A study of the use of individual flash cards compared with the teacher used cards would be interesting in terms of repetitions per pupil per minute and in results obtained from corresponding amounts of time.

Chapter Sixteen

TRANSFER OF TRAINING

MAJOR RESEARCH

Olander, Herbert T.
An Experimental Determination of the Degree of Transfer Between Taught and Untaught Combinations in Simple Addition and Subtraction.
Unpublished Doctor's Dissertation. Department of Education. University of Chicago. 1930.

The purpose of this study was to test the validity of the conclusions substantiated by results of studies of relative difficulty of number combinations - namely, simple number combinations are so many bonds and should be learned as such.

The study involved 1300 children comprising thirty-seven sections taught by twenty-six teachers in grade two B in Detroit, Fordson, and Hamtramck, Michigan. The investigation lasted for seventeen weeks. 55 addition and 55 subtraction facts were taught. In addition the facts included all of the doubles, 7 zero facts, and 6 reversals. The 45 untaught facts were selected so that they fell within the four quarters of increasing difficulty in Clapp's list, also so that twenty-three of them were written with the smaller addend plus the larger. The subtraction facts corresponded to the addition facts, e.g. 5 + 3 = 8 being related to 8-5 = 3.

Groups 1 and 2 were drilled in 110 combinations by

with-generalization and without-generalization methods respectively; Groups 3 and 4 were drilled in 200 combinations by with-generalization and without-generalization methods respectively; Group 5 was without formal instruction in arithmetic for twelve weeks; Group 6 was without formal instruction in arithmetic for seventeen weeks. Groups 1, 2, 3, 4, and 5 were given the same instruction for five weeks. Groups 1, 2, 3, and 4 were given the same material during the next six weeks but Groups 1 and 3 were drilled by a generalization method, Groups 2 and 4 being drilled without generalization. During the last six weeks Groups 1 and 2 continued drill on the 55 addition and 55 subtraction facts while Groups 3 and 4 were given drill on the 90 so-called untaught combinations in addition and subtraction.

Tests including the 100 addition and the 100 subtraction combinations were given at the outset of the experiment and at the end of the fifth, eleventh, and seventeenth weeks. These tests were given by flash cards to reduce counting.

Data are presented in fifty-six tables and seven figures. 111 pages are devoted to the two booklets of directions for teachers.

Results of the four tests for addition taken from tables 14 and 15 are as follows:

		110 Taught		200 Taught	
		Score	Per cent	Score	Per cent
Test 1	Taught	5.41	52.4	5.71	53.5
	Untaught	4.92	47.6	4.97	46.5
	Total	10.33	100.0	10.68	100.0
Test 2	Taught	22.24	54.3	21.72	54.6
	Untaught	18.68	45.7	18.05	45.4
	Total	40.92	100.0	39.77	100.0
Test 3	Taught	30.10	54.8	29.64	54.9
	Untaught	24.79	45.2	24.36	45.1
	Total	54.89	100.0	54.00	100.0
Test 4	Taught	39.57	55.3	36.92	54.4
	Untaught	32.01	44.7	30.95	45.6
	Total	71.58	100.0	67.87	100.0

		Nothing 12 weeks 5 weeks ?		Nothing 17 weeks	
		Score	Per cent	Score	Per cent
Test 1	Taught	5.54	52.5	20.31	54.0
	Untaught	5.01	47.5	17.32	46.0
	Total	10.55	100.0	37.63	100.0
Test 2	Taught	22.47	53.1	23.22	53.3
	Untaught	19.81	46.9	20.31	46.7
	Total	42.28	100.0	43.53	100.0
Test 3	Taught	28.12	54.4	27.88	54.2
	Untaught	23.55	45.6	23.53	45.8
	Total	51.67	100.0	51.41	100.0
Test 4	Taught	30.89	55.1	34.20	55.0
	Untaught	35.20	44.9	27.99	45.0
	Total	56.09	100.0	62.19	100.0

Conclusions are drawn primarily from results of 296 children who had been taught 110 number combinations in addition and subtraction matched with 296 children who had been taught 200, the equality of groups being secured by pairing pupils on (1) initial arithmetic scores, (2) gains in arithmetic scores over a period of five out of seventeen

weeks, and (3) method of teaching.

The author finds almost complete transfer from the 55 combinations to the 45 untaught combinations in each process with slightly less transfer in subtraction than in addition; the groups which were taught all 100 addition and all 100 subtraction combinations made no higher scores on all of the 200 combinations than did the groups which were taught only 55 combinations in each process; teaching only 55 number combinations in each of the processes had no apparent effect on children's ability to solve concrete problems involving certain of the untaught combinations; the absence of any appreciable effect of generalizing; the classes given no instruction in arithmetic during twelve and seventeen weeks learned from a half to less than a third as many number combinations as were learned by children who were being given the regular class instruction.

Comment: The knowledge of number facts possessed by the group without formal instruction leads one to wonder if this study does not point to advantages of informal number.

Overman, J. R.

Experimental Study of Certain Factors Affecting Transfer of Training in Arithmetic.

Warwick and York, 1931. pp. 235.

This experiment carried out in 1927-1928 in twenty second grades in Toledo, Ohio, was repeated in sixteen second grades in Toledo, one in Bowling Green, and fourteen in Findlay, Ohio.

The author states that his purpose was to study the

effects of instruction on three types of examples in two-place addition upon pupil's ability to handle closely related types in three-place addition and in two and three-place subtraction.

The grades were divided into four different groups, a different method of instruction being employed with each of the groups. Group A was shown how to perform the process with no generalization or rationalization. This became the control group. Group B was helped form the generalization that the numbers must always be written so as to keep the right hand column straight. Group C was taught the rationalization method, in this case discussing the principle that ones can only be added to ones and tens to tens. Group D was taught both the generalization and rationalization as described above.

A test series consisting of three tests was given to each group four times; at the beginning, at the end, and twice during the experiment.

Data are fully presented in forty-four tables and fifteen figures.

The author's findings are as follows:

1. The effect of the instruction and practice given on certain specific types of examples was not confined to those types but spread or transferred to related types. Some spread occurred in the cases of 99.1 per cent of all pupils. The mean transfer, for the group taught by the most favorable method, was 72.4 per cent of complete transfer. Of this group 15.2 per cent obtained 100 per cent transfer.

2. Some of the types of examples given on the test seemed to be so closely related to the specific types that were taught that the transfer took place automatically without any conscious generalization on the part of the pupils or else the generalization was so simple that the majority of the pupils were able to make it without assistance. At any rate there was little or no difference in the effectiveness of the different methods of teaching in producing transfer to examples of these types.

3. On the types of examples where the generalization and rationalization would naturally be expected to be most effective, namely on those involving the placing of numbers having different numbers of digits, there were fairly large and reliable differences between the methods. On examples of this type the generalization alone increased the transfer by 45.1 per cent, the rationalization alone increased it by 15.5 per cent, and the generalization and rationalization combined increased it 36.9 per cent. "It appears therefore" says Overman on page 121, "that aid in generalizing the process was an effective method of increasing transfer with pupils in the second grade."

4. There were no large or reliable differences between the two sexes with respect to transfer, although the generalization may possibly have been slightly more effective with the boys than with the girls.

5. In general there was an increase in the amount of transfer with an increase in mental age.

6. The generalization and rationalization were most effective

in producing transfer with pupils in the highest third of the group with respect to mental age. No certain differences were found between the methods with pupils in the two lower thirds, although the evidence indicates that the generalization method was probably somewhat superior.

7. The relative amount of transfer to the different types of examples was not haphazard but could be accounted for as a result of the intrinsic difficulty of the different types and their degree of similarity or dissimilarity to the specific types that were taught.

8. The greatest transfer to any given type of example occurred on the next testing after the pupils had been taught the type most nearly related.

9. The rationalization reduced the amount of transfer on the second and third testings. On the fourth testing, rationalization produced as much transfer as the first method, neither rationalization nor generalization, and the last method, both rationalization and generalization, produced as much as generalization alone.

A bibliography of 38 references is followed by a 76 page appendix containing tables, tests, and directions to teachers.

MINOR RESEARCH

Winch, W.H.
Accuracy in School Children. Does It Transfer?
Journal of Educational Psychology 1:557-589. December 1910.

Children from four schools who took part in this experiment were divided on the basis of several tests. These tests were marked on accuracy of reasoning, not the numerical answer.

During the experiment one group practiced arithmetic and the other group practiced some other subject. The final test was given to both groups.

Data are recorded in fifteen tables.

Specific results show that it was possible to improve the accuracy of numerical computation without any certainty of thereby improving reasoning.

Winch, W.H.
Further Work on Numerical Accuracy in School Children.
Does Improvement in Numerical Accuracy Transfer?
Journal of Educational Psychology 2:262-271. May 1911.

Seventy-two boys were divided into two groups on the basis of six reasoning tests. One group practiced arithmetic and the other practiced drawing for ten practice periods. Six tests in arithmetic reasoning were given at the end of the practice periods.

Results are set forth in six tables.

Great improvement was found in arithmetic computation but no improvement in arithmetic reasoning. In both groups the improvement was practically equal in arithmetic reason-

ing. There was a high positive correlation between numerical computation and arithmetic reasoning but it does not involve such community of function that improvement in one produces improvement in the other.

Starch, D.
Transfer of Training in Arithmetical Operations.
Journal of Educational Psychology 2:306-310. May 1911.

Eight observers worked for fourteen days on mental multiplication. Six tests - adding fractions, adding two three-place numbers, subtracting three-place numbers, multiplying a four-place number by a one-place number, dividing a three-place number by a one-place number - were given besides two memory span tests.

Results are recorded in three tables.

The average per cent gained by the eight was 50. The average, exclusive of the memory span, was 49 for the trained and 20 for the untrained.

From introspection the observers said that the practice had increased the ability to apprehend and hold numbers in mind, and to visualize arithmetic operations.

Knight, F. B. and Setzafandt, A. O. H.
Transfer Within a Narrow Mental Function.
Elementary School Journal 24:780-787. June 1924.

Two groups of ability as nearly equal as possible were formed but the number and grade are not mentioned.

Group A worked with denominators 2, 3, 4, 5, 6, 7,

8, 9, 10, 12, 14, 15, 16, 18, 21, 24, 28, 30. Group B used only 2, 4, 6, 8, 9, 10, 12, 16, 24.

Two tests were given, the first including the denominators practiced by both groups, the second containing those denominators practiced by group A but not by group B.

Six tables present the data.

There was a slight loss for both groups in the test containing the fewer denominators but this loss was proportional. There was a high degree of transfer from the familiar to the unfamiliar. There were two transfers (1) the common denominator idea and (2) the knowledge of relations between whole numbers. The transfer was mechanical and unconscious, the children not realizing that they hadn't used 3, 5, 7, etc., in drills.

The authors conclude that transfer is possible to the extent to which skills exist to transfer.

Beito, E. A. and Brueckner, Leo J.
A Measurement of Transfer in the Learning of Number Combinations.
Twenty -ninth Yearbook of the National Society for the Study of Education : 569-587. 1930.

Three two B grades containing 32, 33, and 28 children were involved in this study. Thirty-six addition facts which are reversible were arranged, twelve on a card, each card containing some easy, some average, and some hard according to Smith's order of relative difficulty. Studying the facts with their answers on the reverse side of the card was called the drill-card method. The experiment lasted three weeks

each class studying one card for a week. The 12 facts on Card 1 were studied by the usual methods plus the drill-card method, Card 2 was studied by the usual method, Card 3 by the drill-card method only. Tests were given on the facts and their reverses.

Four tables give results on the pre-test, daily and final tests for the three classes, and the total results for the 93 children.

The authors conclude that the reverse form is learned concomitantly at least as completely as the direct form on any mental level; that the bond formed in learning the direct form carries over almost completely to the reverse form; that the most efficient method was the teacher's usual method together with the use of drill cards.

SUMMARIES

Overman, J. R.

An Experimental Study of the Effect of the Method of Instruction on Transfer of Training in Arithmetic.

Elementary School Journal 31:183-190. November 1930.

This article reviews the major research by Overman reported in this chapter.

The problem was to determine which method gave most transfer from two-place addition to three-place addition and to subtraction, the methods being (1) no generalization or rationalization, (2) generalizing the procedure, (3) rationalizing the procedure, and (4) generalization and rationalization together.

Results show that the effect of instruction and practice spread to related types. The generalization process increased transfer by 45.1%, the rationalization by 15.5%, and the generalization and rationalization together by 36.9%.

Olander, Herbert T.
Transfer of Learning in Simple Addition and Subtraction I.
Elementary School Journal 31:358-369. January 1931.

This article and the one below review the major research by Olander reported in this chapter.

Olander, Herbert T.
Transfer of Learning in Simple Addition and Subtraction II.
Elementary School Journal 31:427-437. February 1931.

This article and the above review the major research by Olander reported in this chapter.

SUMMARY OF CHAPTER SIXTEEN

Type of Study	Author and Date	Nature of Study	Number of Subjects
Major Research	Olander, H.T. 1930	Experiment to determine degree of transfer between taught and untaught combinations in simple addition and subtraction.	1300
	Overman, J.R. 1931	Effect of instruction in two-place addition upon ability to handle three-place addition and two and three-place subtraction.	50 second grades

Type of Study	Author and Date	Nature of Study	Number of Subjects
Minor Research	Winch, W.H. 1910	Experiment to determine the effect upon reasoning of improvement in computation.	4 schools
	Winch, W.H. 1911	Further experiment to determine effect of improved computation upon reasoning.	72
	Starch, D. 1911	Experiment to determine effect of mental multiplication upon other phases of computation.	8
	Knight, F.B. and Setzafandt, 1924	Experiment to determine effect of teaching addition of fractions with denominators 2,4,6, 8,9,10,12,16, and 24 upon ability to add fractions with denominators 2,3,4,5,6,7,8,9,10,12,14, 15, and 16.	Number not given
	Beito, E.A. and Brueckner, L.J. 1930	An experiment to determine the effect of learning addition facts in their direct form upon the reverses of these facts.	93
Summaries	Overman, J.R. 1930	Review of major research by this author reported earlier in this chapter.	
	Olander, H.T. 1931	Two articles in successive issues reviewing the major research by the author which occurs earlier in this chapter.	

CONCLUSIONS

Two recent major researches relating to transfer are by Olander and by Overman. In the former the author finds almost complete transfer from the 55 "taught" combinations, in addition and subtraction, to the 45 "untaught" combinations. One may reasonably question this amount of transfer in view of the data occurring in the study. The differences in achievement in the experimental groups and in a control group having no teaching for seventeen weeks leads one to

question whether outside influences rather than transfer may explain the knowledge of the "untaught" combinations. The latter study, by Overman, deals with the effect of instruction in two-place addition upon three-place addition and two-and three-place subtraction. The author finds more transfer when using the generalization method, that numbers must always be written so as to keep the right hand column straight. One may wonder what transfer really means and if this effect on material so closely related is transfer.

Early minor researches by Winch in 1911 and 1912 and by Starch in 1911 examine respectively the effect of improved computation upon reasoning and effect of mental multiplication upon other phases of computation. Later minor researches include that by Knight and Setzafandt to determine the effect of teaching addition of fractions with certain denominators upon the ability to add fractions with other denominators, in which they find a high degree of transfer and conclude that transfer is possible to the extent to which skills exist to transfer. From a study of the effect of learning addition facts in their direct form upon the reverses of these facts Beito and Brueckner find that the reverse form is learned concomitantly at least as completely as the direct form.

A summary of the Overman study is found in the Elementary School Journal for November 1930, and in the January 1931 and February 1931 issues of this magazine are found summaries of the Olander study.

SUMMARY AND CONCLUSIONS

The following table shows the number of studies of each class occuring in each of the chapters of the thesis:

	Major Research	Minor Research	Library Summary	Discussion
The Development of the Number Concept	4	1	1	2
The Nature of Primary Number-Formal or Informal	4			
Reading and Writing of Numbers	2	2	1	
The Relative Difficulty of the Number Combinations	5	2	4	
Addition	4	8	2	4
Subtraction	5	4	3	10
Multiplication	1	3	1	4
Division		3	1	6
Common Fractions	5	3	1	12
Decimal Fractions		3	1	3
Percentage		2	2	1
Denominate Numbers		1		
Checking Computation		1		2
Motivation	3	3	2	
Drill	2	33	2	6
Transfer of Training	2	5	3	
Totals	37	74	24	50

1. Limitations.

The writer is fully aware of the many weaknesses in this study but is likewise aware that the time which would be required for greater perfection would be out of proportion to the value of the perfected work. Some of the major difficulties in a study of this nature are:

Content. The writer is sure that many worthwhile materials have not been included because of ignorance of their existence, difficulty of access to them, and sheer lack of time to prepare summaries under existing conditions.

Classification. Without objective criteria the attempt to classify researches as major or minor has been very unsatisfactory. In making a study of this kind again the writer would abandon any attempt at such classification until research specialists had provided definite standards for judgment.

Scope. If in the light of recent experience this work were being repeated the writer would limit the scope of the individual summaries to those published within the last fifteen years or even perhaps the last ten years indicating briefly earlier studies. This thesis has become bulky almost beyond reason because of an ambition to explore the whole field.

Viewpoint. The habit of viewing newly acquired information in the light of its use in modifying one's own teaching practice is bound to give a limited viewpoint although a practical one. In trying to help college students to learn to distinguish between conclusions based on opinion

and conclusions based on scientific research it is desirable that the helper should be able to send these students to materials of both kinds. To be able to quote authorities when presenting new viewpoints and to be able to answer explicitly the question "What has been done in this field?" makes a task of the size of this thesis of greater value to the writer than to the chance reader.

Lack of uniformity in treatment of material. The critic may find that some studies have been reported in unwarranted detail while others have not received the attention due them. Unpublished theses and dissertations have been reported rather fully because of the impossibility of frequent reference to them. Studies easily obtained and those better known to students of arithmetic received perhaps less detailed attention. No doubt the length of time during which articles were being summarized will account for some lack of uniformity.

2. Values derived from the preparation of this thesis.

In spite of the limitations listed above and of other existing weaknesses the preparation of this thesis has made possible the following results:

An acquaintance with the extent and nature of research in each of the special subject matter fields and in the general fields of motivation, drill, and transfer.

Acquaintance has been made with the work of such specialists as Buckingham, Brueckner, Buswell, Knight, Morton, G.C.Myers, Osburn and others, resulting in a knowledge of the viewpoint and treatment of material likely to

exist in articles prepared by these specialists and by graduate students working under their direction.

Acquaintance has been made with the offerings of a wide range of educational magazines resulting in a knowledge of the kind of study likely to be contained in each magazine. The magazines likely to contain the writings of certain authors can be told to some extent.

A desire has grown out of this work to continue this acquaintance with researches, authors, and magazines by keeping up to date on current publications.

There has resulted an appreciation of the need for investigation of a controlled nature. An overwhelming majority of the studies reported have investigated quantity and quality of learning under existing conditions instead of comparing results of learning under two or more sets of controlled conditions.

3. Forward steps in arithmetic research.

Since quite comprehensive tabular and verbal conclusions are contained at the end of each chapter it seems desirable here to note more especially those phases of arithmetic in which research is lacking or insufficient.

1. The student of arithmetic methods cannot help being impressed with the great degree of variation in the choice of the topics chosen, in the number of subjects involved, in the methods employed, in the duration of the experiments, and in the treatment of data in the published studies and available theses and dissertations relating to this field of education. Any attempt to

classify studies of this nature must to an undesirable extent be subjective because of the lack of objective criteria. Even an attempt to use the tentative criteria devised in seminar work among college students fails because of the scarcity of experimentation making use of control groups, and because of the prevalence of reports in the nature of field studies. From the viewpoint of use of published research with college students in education courses it would be helpful to have definite criteria to apply to studies under consideration. What number of subjects, what treatment of experimental material, what degree of control, what manner of equating groups for control, what statistical treatment of results, what period of duration of an experiment, make it possible to feel that findings are conclusive rather than tentative? From the viewpoint of the student who is interested to do a piece of original research in the field of arithmetic methods these same problems arise if that student is desirous of doing work that is really worth while. My first conclusion, therefore, is the presence of a need for definite research criteria formulated by those educators who are preparing and directing research.

2. It has been difficult to differentiate satisfactorily between studies relating to the development of number concepts and those relating to the nature of the teaching in primary grades. The studies in Chapter One dealing with the first of these two topics, were chosen because they attacked the problem from the psychological angle

or because they were studies of number concepts gained before entering first grade. The studies reported by Miss Josephine Maclatchy and by Mrs. Sophie Court showing the quantity of number known by pre-school children lead to the suggestion that observations be made of pre-school children to determine the manner in which these number concepts were gained. Greater use of inventory testing as suggested by Woody would add further to the work by the authors mentioned above.

3. Since the contacts of the writer with both students and experienced teachers indicate that "incidental", "informal", and "formal" have many different meanings a clear discussion of the nature of each seems worth while.

4. While there are without doubt many progressive school systems in which informal number has replaced formal number in the first or first two grades there are as many systems teaching formal number from the very beginning of first grade. There are but few studies which present results of informal work. Connor's suggestion that experiments be carried on comparing groups from kindergarten through ninth grade to test results of informal and formal teaching in grades one to three, is worthy of consideration.

5. Terry's research regarding the reading of numbers by adults is outstanding from the viewpoint of the mechanics of numeral reading. In the field of problem solving much has been done to compel more careful read-

ing and interpretation of numbers. No studies reaching the attention of the writer indicate the manner in which children treat numerical references occurring in general reading and requiring no check-up. College students of education admittedly "skip" such numerical references. Do children likewise skip them or do they interpret them? If the latter course is taken what value is derived in terms of greater information and appreciation?

6. It has long been customary for some teachers to strive for the reading of whole numbers without the use of "and" because of the desirability of using this expression in reading the decimal point in numbers containing decimal fractions. How generally are public school teachers emphasizing this through example and training? Is it desirable that the practice be observed or should we give up the struggle and accept the manner of reading numbers in use by the general public? No research or discussion on this point came to the attention of the writer.

7. In several studies classes of errors were explained as resulting from interference from a skill previously learned. Since the emphasis in present day teaching is prevention of errors would it not be helpful to have investigations and analyses of subject matter to discover possibilities of interference so that a part of teaching technique may be the anticipation of and prevention of interference errors?

8. Studies of relative difficulty have in general covered all processes although addition and multiplication have received most attention. Early investigations, notably Holloway's, used fewer than 100 primary facts in addition and multiplication. The test technique used by Holloway, Clapp, and Washburne and Vogel has been abandoned by Knight and his collaborators in favor of the counting of repetitions made by each subject for each fact. It is a question whether difficulty due to faulty training has been differentiated from inherent difficulty in number facts. Perhaps some further type of research will be devised for handling this problem. In the meantime recognition of 100 facts in each addition, subtraction, and multiplication and 90 in division is producing more comprehensive drill with greater accuracy.

9. The problem in addition receiving most attention is that of upward versus downward adding. Both of Buckingham's studies contain arguments for downward addition not justified by the data. No doubt Manuel's suggestion that both must be taught and that the order in which the teaching occurs makes little difference, is the most sensible view.

10. Only one study reports the comparison of results from teaching addition and subtraction primary facts together and teaching them separately. Although the results point to better results if the facts are taught together the author, Buckingham, suggests more extensive

investigation of this problem.

11. Although counting is frequently mentioned as a type of error to be avoided in addition, no studies either research or discussion show how the establishing of this habit is to be prevented. Probably much help could be given if experiences in preventing counting were made available through publication.

12. The inventory testing described in the studies by Miss Maclatchy showing the amount of addition known to children entering school might be gradually extended to cover all of the primary facts instead of a few.

13. Of the nineteen articles relating to subtraction twelve have to do with the method to be used in subtracting. In these experiments results seem slightly in favor of the take-away-carrying or equal additions method. The lack of uniformity in terminology makes interpretation of results difficult. As pointed out in Chapter Six, Austrian may mean addition-borrowing or addition-carrying. It is to be hoped that someone will publish a discussion calling attention to the desirability in future studies of common terminology. The twelve studies referred to cover the period from 1919 to 1931. Is it not reasonable to expect that if there were great advantages of one method over another these advantages would have been discovered conclusively in the last decade of experimentation. If there are to be further studies it is to be hoped that a controlled experiment of considerable size will investigate not only the

method in use but the effects of teaching to equated groups the four commonly used methods. As has been suggested this experiment should involve subtraction of fractions and mixed numbers as well as subtraction of integers. If this does not seem worth while a comprehensive investigation of the subtraction method in most common use in a large part of the country would show the method most popular with teachers and administrators and this method might be adopted in the interests of uniformity.[1]

14. It is recognised by some authors of drill services and text-books that 175 upper decade subtraction facts are needed for short division. It seems common procedure to teach these as a part of the subtraction task although they are not used until primary uneven division facts are taught. A controlled experiment would be of interest which would compare results from teaching these facts with other parts of subtraction with results from teaching them when needed in division.

15. Little valuable experimentation has been performed in multiplication. An error study by Clemens and Neubauer and the study of relative difficulty of primary multiplication facts by Norem and Knight are the most worth while of the multiplication studies. The writer finds that students in a teachers college are relatively unaffected by an overwhelming list of arguments for teaching primary multiplication facts in unrelated groups instead of in tables. Therefore, it would be most grat-

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[1] Such a study was completed in June 1932 by Blanche M. Allen. Master's Thesis. School of Education. Boston University.

ifying to be able to refer to a scientific research showing results obtained from both plans carried on under controlled conditions.

16. There is some tendency to replace the expression "times" as in "two times six" by the expression "two sixes" or "six twos". The abuses of the expression "times" with which all teachers are acquainted, which grow out of the lack of significance of its meaning, suggest the adoption of the expression which carries no question as to its meaning.

17. No major researches have been made in division, an interesting observation in view of the alleged difficulty of the teaching of division. Two recent lines of interest in division have been (1) the use of the long division form in teaching division by one-digit numbers, investigated by Miss Lenore John, and (2) the use of the apparent-quotient and increase-by-one rules for estimating quotient figures, discussed by Grossnickle. On the first problem results point to greater accuracy when division by one-digit divisors is taught by the long division form but further investigation on a larger scale is needed. The comparison of the apparent-quotient and increase-by-one rules is at present theoretical. Grossnickle has prepared careful analyses of possible division examples showing cases in which each rule functions. With this preliminary analysis investigation based on results of classroom teaching is needed.

18. In the field of common fractions the outstanding type of study has been the analysis of errors. Studies of this kind have been made by Kallom, Brueckner, James, Kee, and Morton. A study by Miss Polkinghorne of the fraction concepts of primary school children is another worthy contribution. The remaining articles include argumentative discussions by Knight and by Brueckner concerning "unit skills" and "total situations" as bases for gradation of subject matter of common fractions. Osburn further suggests "job analysis" as the basis for organization. In much of this analysis there is a tendency to disregard social utility. A needed research in common fractions is a drill service based on socially useful computation.

19. But little interest has been shown in decimal fractions, no major researches appearing in this field. Brueckner's error study is no doubt the most helpful study. Drushel's investigation showing the value of placing the point in division of decimals by making the divisor a whole number instead of by pointing off a number of places in the answer equivalent to the number of places in the dividend less the number in the divisor, has no suggestion for the present although doubtless of value in 1917 when reported. Wright's discussion of the same phase loses value because the material is so far beyond social usage. Arleigh's suggestion for the use of cartoons, to make vivid the function of the decimal point, is thought-provoking but carries a question of

the need for this visual aid if meaning is thoroughly taught. There is mention by Miss Sarah Tracy in the Mathematics Teacher for March 1925 of the attempt in Newton to teach the placing of the decimal point by inspection instead of by rule. The lack of judgment in placing the decimal point, exhibited by many college students, raises a question whether or not better results could be obtained if the computation involving multiplication and division of decimal fractions were limited to socially useful content with the decimal point placed by inspection. In this connection experimentation would be worth while to discover the effect of using for a part of the drill materials examples in which the computation has been performed calling only for the placing of the point.

20. The study offering most promise in percentage is that by Miss Wilker, studying the effect of the teaching method on results obtained in cases 2 and 3 in percentage. The original study being unavailable the writer had to be satisfied with a summary. Another summary, by Miss Mahoney, reports unit skills in percentage. Error studies by Kircher and by Edwards point to a lack of understanding of what percentage really means. It is to be hoped that studies may soon be made determining the best ways in which comprehension of percentage principles may be gained by children. The effect of real understanding of percentage on computation involving percentage would complete the study.

21. Until recently little interest has been shown in denominate numbers. The studies by Wilson and by others in Education for February 1932 show this recent interest to be in the curricular rather than the methods phases. One study, by Springer, reports better results obtained through problem solving with reference to tables as units were needed than through the traditional memorizing of tables followed by problem solving. A study on a larger scale using measuring units needed in life situations might favorably affect teaching practice.

22. In this day of emphasis on 100% accuracy in computation there may arise a corresponding interest in checking methods. The only research made of the value of checking, that by Clark and Vincent, shows greater accuracy resulting from checking. Further studies might be made to determine the degree of accuracy resulting from checking in each of the fundamentals, also to compare checking methods as to ease of adoption and as to results.

23. The outstanding research in motivation is that by Wilson, in which the types of motivation reported used by teachers are classified to include real and idealized life situations, real games, game devices, and devices. This classification seems worth while in view of the custom of calling all motivating activities "games", as so many authors do. Steinway's report of an investigation to determine the value of games leads to the suggestion that comparison be made of the effect of the use of real games instead of the so commonly used de-

vices. Also desirable would be controlled investigation of the effect of making real games a part of the teaching technique instead of using the game or device as it more often is, as a reward following flash card or other drill.

24. In the study by Panlasigui and Knight the conclusions do not point to strong motivation resulting from the use of standards and graphs of pupil's own scores. Can this be because so much teaching stresses competition between groups and between individuals? Aren't the results of the experiment affected by the attitudes acquired by the children in all of the teaching preceding the experiment as well as by the immediate teaching during the experiment? Would different results be found if an experiment were controlled where, from the first days in school, children in equated groups experienced different motives?

25. The topics receiving attention in drill have been the value of drill, the distribution of drill periods, the losses due to summer vacations, the value and the means of individualizing drill, the effect of emphasis on accuracy and on speed, motivation of drill, distributed and bunched drill, persistence of errors, comparison of oral and written drill, and several other phases not of major importance. With the present emphasis on individual drill, further investigation might determine the most effective kinds of individual drill. A comparison of the use of individual drill plans and

of group drill might lead to less chorus response unless value were found in such practice. For example a comparison of the results in terms of repetitions per pupil per minute for a given period of time in the use of individual flash cards and teacher used flash cards, would yield practical information.

26. The studies of transfer are interesting especially those in regard to the transfer from "taught" to "untaught" combinations and transfer from direct to reverse facts. However, these do not seem entirely convincing in view of the results of the studies of the relative difficulty of number facts in which several studies found little relation between the direct fact and its reverse. Is transfer always unconscious as Knight and Setzafandt found it to be in their fraction study or may the conscious effort to find elements in the known which offer solution of the unknown a type of transfer? The absence of this latter kind of transfer is a challenge to the educator in any field.

27. In the examination of nearly 200 studies bearing on arithmetic methods the writer has been impressed with the prevalence of two types of materials. There are many investigations based upon tests administered to few or many subjects. There are likewise many studies reporting theoretical analysis of "unit skills" and the like. While studies of both of these types have a place especially in a pioneer period of research it is to be hoped that we may have an increasing number of compre-

hensive investigations under controlled conditions.

28. In the discovery of those studies which relate to methods a tremendous number of curricular studies were encountered showing that field to be undoubtedly the investigator's prime interest. Very little research has dealt specifically with the method of teaching the subject matter phases of arithmetic. Comparison of differing methods and analysis of errors resulting from methods in general have been more common. It would seem that the methods course in a teacher training institution is no longer a passing on of a "bag of tricks", as critics of such courses have been accustomed to say, but requires a knowledge of curriculum construction, of reasonable organization of facts and process steps, of principles of motivation and of drill, and the place and use of activities as well as a knowledge of problem solving and remedial work not included in this thesis.

tensive investigations under controlled conditions.

40. In the discovery of those phases which belong to methods a thorough survey of curricular offerings concentrated strictly that fails to be undertaken. the investigator's prime interest. Very little research has dealt specifically with the method of teaching the subject matter phases of mathematics. Comparison of differing methods and analysis of errors resulting from methods in general have been more common. It would seem that the methods course in a teacher training institution is no longer a passing on of a "bag of tricks", as critics of such courses have been accustomed to say, but requires a knowledge of curriculum construction, of reasonable organization of facts and process steps, of principles of motivation and of drill, and the place and use of motivation as well as a knowledge of problem solving and remedial work such included in this thesis.

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